Dynamics

Dynamics includes:-

1- Kinematics: study of the geometry of motion.

Relates displacement, velocity, acceleration, and time ***without reference*** to the cause of motion.

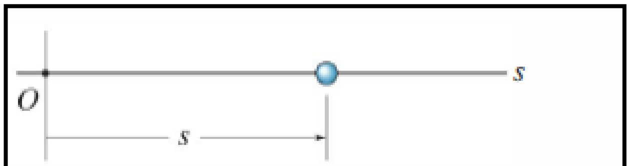
Kinetics: study of the relations existing between the forces acting on a body, the mass of the body, and the motion of the body. Kinetics is used to predict the motion caused by given forces or to determine the forces required to produce a given motion.

Kinematics of Particles

Mechanics is a branch of the physical sciences that is concerned with the state of rest or motion of bodies subjected to the action of forces.
Engineering mechanics is divided into two areas of study, namely, statics and dynamics. Statics is concerned with the equilibrium of a body that is either at rest or moves with constant velocity. Here we will consider dynamics, which deals with the accelerated motion of a body. The subject of dynamics will be presented in two parts: kinematics, which treats only

the geometric aspects of the motion, and kinetics, which is the analysis of the forces causing the motion. To develop these principles, the dynamics of a particle will be discussed first, followed by topics in rigid-body dynamics in two and then three dimensions.

- Position. The straight-line path of a particle will be defined using a single coordinate axis s, Fig. 1a. The origin 0 on the path is a fixed point, and from this point the position coordinate s is used to specify the location of the particle at any given instant. The magnitude of s is the distance from 0 to the particle, usually measured in meters (m) or feet (ft), and the sense of direction is defined by the algebraic sign on s. Although the choice is arbitrary, in this case s is positive since the coordinate axis is positive to the right of the origin. Likewise, it is negative if the particle is located to the left of O. Realize that position is a vector quantity since it has both magnitude and direction. Here, however, it is being represented by the algebraic scalar s since the direction always remains along the coordinate axi.

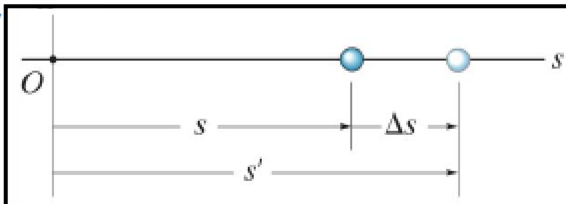


Position
(a)

- Displacement. The displacement of the particle is defined as the change in its position. For example, if the particle moves from one point to another, Fig. b, the displacement is

$$\Delta s = s' -$$

In this case Δs is positive since the particle's final position is to the right of its initial position, i.e., S’ > s. Likewise, if the final position were to the left of its initial position Δs would be negative



Displacement

(b)

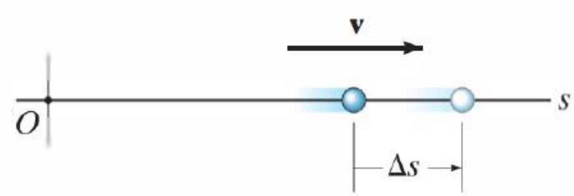
Velocity. If the particle moves through a displacement Δs during the time interval Δt, the average velocity of the particle during this time interval is

$$v_{avg} = \frac{\Delta s}{\Delta t}$$

If we take smaller and smaller values of Δt, the magnitude of Δs becomes smaller and smaller. Consequently, the instantaneous velocity is a vector defined as

$$v = \frac{ds}{dt}$$

Since Δt or dt is always positive, the sign used to define the sense of the velocity is the same as that of Δs or ds. For example, if the particle is moving to the right, Fig. c, the velocity is positive; whereas if it is moving to the left, the velocity is negative



Velocity

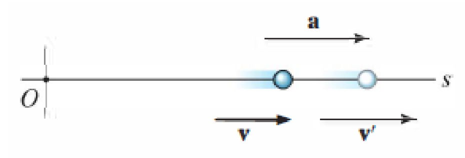
(c)

$$(v_{sp})_{avg} = \frac{s_T}{\Delta t}$$

- Acceleration. Provided the velocity of the particle is known at two points, the average acceleration of the particle during the time interval Δt is defined as

$$a_{avg} = \frac{\Delta v}{\Delta t}$$

Here ΔV represents the difference in the velocity during the time interval Δt, i.e, Δv = v' - v, Fig. e.



Acceleration

(e)

The *instantaneous acceleration* at time t is a vector that is found by taking smaller and smaller values of Δt and corresponding smaller and smaller values of Δv, so that $a = \lim_{\Delta t \to 0} (\Delta v / \Delta t)$, or

$$a = \frac{dv}{dt}$$

$$a\,ds = v\,dv$$

Constant Acceleration, $a = a_c$. When the acceleration is constant, each of the three kinematic equations $a_c = dv/dt$, $v = ds/dt$, and $a_c\,ds = v\,dv$ can be integrated to obtain formulas that relate a_c, v, s, and t.

Velocity as a Function of Time. Integrate $a_c = dv/dt$, assuming that initially $v = v_0$ when $t = 0$.

$$\int_{v_0}^{v} dv = \int_0^t a_c\,dt$$

$$v = v_0 + a_c t$$

Constant Acceleration

Position as a Function of Time. Integrate $v = ds/dt = v_0 + a_c t$, assuming that initially $s = s_0$ when $t = 0$.

$$\int_{s_0}^{s} ds = \int_0^t (v_0 + a_c t)\,dt$$

$$s = s_0 + v_0 t + \tfrac{1}{2} a_c t^2$$

Constant Acceleration

- Velocity as a Function of Position

$$\int_{v_0}^{v} v\,dv = \int_{s_0}^{s} a_c\,ds$$

$$v^2 = v_0^2 + 2a_c(s - s_0)$$

Constant Acceleration

The algebraic ... above three equations, are determined ... s axis as indicated by the arrow written at the left of each equation. Remember that these equations are useful *only when the acceleration is constant and when* $t = 0$, $s = s_0$, $v = v_0$. A typical example of constant accelerated motion occurs when a body falls freely toward the earth. If air resistance is neglected and the distance of fall is short, then the *downward* acceleration of the body when it is close to the earth is constant and approximately $9.81\ \text{m/s}^2$ or $32.2\ \text{ft/s}^2$. The proof of this is given in Example 13.2.

Important Points

- Dynamics is concerned with bodies that have accelerated motion.
- Kinematics is a study of the geometry of the motion.
- Kinetics is a study of the forces that cause the motion.
- Rectilinear kinematics refers to straight-line motion.
- Speed refers to the magnitude of velocity.
- Average speed is the total distance traveled divided by the total time. This is different from the average velocity, which is the displacement divided by the time.
- A particle that is slowing down is decelerating.
- A particle can have an acceleration and yet have zero velocity.
- The relationship $a\,ds = v\,dv$ is derived from $a = dv/dt$ and $v = ds/dt$, by eliminating dt.

EXA M P L E -1-

The car in Fig. below moves in a straight line such that for a short time its velocity is defined by v = (3t + 2t) ft/s , where t is in seconds. Determine its position and acceleration when t = 3 s. When t = 0, s = 0.

SOLUTION

Coordinate System. The position coordinate extends from the fixed origin O to the car, positive to the right.

Position. Since $v = f(t)$, the car's position can be determined from $v = ds/dt$, since this equation relates v, s, and t. Noting that $s = 0$ when $t = 0$, we have*

$$(\overset{+}{\rightarrow}) \qquad v = \frac{ds}{dt} = (3t^2 + 2t)$$

$$\int_0^s ds = \int_0^t (3t^2 + 2t)dt$$

$$s\Big|_0^s = t^3 + t^2\Big|_0^t$$

$$s = t^3 + t^2$$

When $t = 3$ s,

$$s = (3)^3 + (3)^2 = 36 \text{ ft} \qquad \textit{Ans.}$$

Acceleration. Since $v = f(t)$, the acceleration is determined from $a = dv/dt$, since this equation relates a, v, and t.

$$(\overset{+}{\rightarrow}) \qquad a = \frac{dv}{dt} = \frac{d}{dt}(3t^2 + 2t)$$

$$= 6t + 2$$

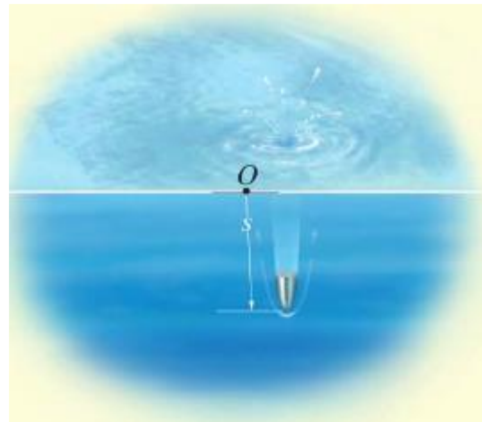
When $t = 3$ s,

$$a = 6(3) + 2 = 20 \text{ ft/s}^2 \rightarrow \qquad \textit{Ans.}$$

EXA M P L E -2- A small projectile is fired vertically downward into a fluid medium with an initial velocity of 60 m/s. Due to the drag resistance of the fluid the projectile experiences a deceleration of a = (-0.4v3) m/s2, where v is in m/s. Determine the projectile's velocity and position 4 s after it is fired.

SOLUT I O N

Coordinate System. Since the motion is downward, the position coordinate is positive downward, with origin located at 0, Fig



Velocity. Here $a = f(v)$ and so we must determine the velocity as a function of time using $a = dv/dt$, since this equation relates v, a, and t. (Why not use $v = v_0 + a_c t$?) Separating the variables and integrating, with $v_0 = 60$ m/s when $t = 0$, yields

$$(+\downarrow) \qquad a = \frac{dv}{dt} = -0.4v^3$$

$$\int_{60\text{ m/s}}^{v} \frac{dv}{-0.4v^3} = \int_0^t dt$$

$$\frac{1}{-0.4}\left(\frac{1}{-2}\right)\frac{1}{v^2}\Bigg|_{60}^{v} = t - 0$$

$$\frac{1}{0.8}\left[\frac{1}{v^2} - \frac{1}{(60)^2}\right] = t$$

$$v = \left\{\left[\frac{1}{(60)^2} + 0.8t\right]^{-1/2}\right\} \text{ m/s}$$

Here the positive root is taken, since the projectile will continue to move downward. When $t = 4$ s,

$$v = 0.559 \text{ m/s}\downarrow \qquad \textit{Ans.}$$

Position. Knowing $v = f(t)$, we can obtain the projectile's position from $v = ds/dt$, since this equation relates s, v, and t. Using the initial condition $s = 0$, when $t = 0$, we have

$$(+\downarrow) \qquad v = \frac{ds}{dt} = \left[\frac{1}{(60)^2} + 0.8t\right]^{-1/2}$$

$$\int_0^s ds = \int_0^t \left[\frac{1}{(60)^2} + 0.8t\right]^{-1/2} dt$$

$$s = \frac{2}{0.8}\left[\frac{1}{(60)^2} + 0.8t\right]^{1/2}\Bigg|_0^t$$

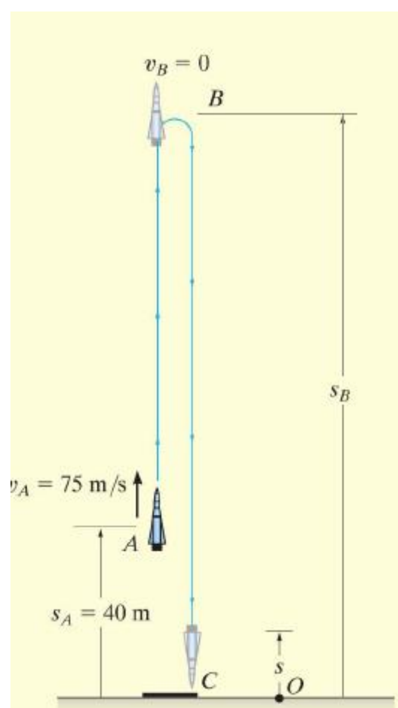
$$s = \frac{1}{0.4}\left\{\left[\frac{1}{(60)^2} + 0.8t\right]^{1/2} - \frac{1}{60}\right\} \text{ m}$$

When $t = 4$ s,

$$s = 4.43 \text{ m}$$ *Ans.*

EXA M P L E -3-

During a test a rocket travels upward at 75 m/s, and when it is 40 m from the ground its engine fails. Determine the maximum height SB reached by the rocket and its speed just before it hits the ground. While in motion the rocket is subjected to a constant downward acceleration of 9.81 m/s2 due to gravity. Neglect the effect of air resistance.



SO LUTI O N:-

Coordinate System. The origin 0 for the position coordinate s is taken at ground level with positive upward, Fig

Maximum Height. Since the rocket is traveling *upward*, $v_A = +75\text{m/s}$ when $t = 0$. At the maximum height $s = s_B$ the velocity $v_B = 0$. For the entire motion, the acceleration is $a_c = -9.81 \text{ m/s}^2$ (negative since it acts in the *opposite* sense to positive velocity or positive displacement). Since a_c is *constant* the rocket's position may be related to its velocity at the two points A and B on the path by using Eq. 12–6, namely,

$$(+\uparrow) \qquad v_B^2 = v_A^2 + 2a_c(s_B - s_A)$$

$$0 = (75 \text{ m/s})^2 + 2(-9.81 \text{ m/s}^2)(s_B - 40 \text{ m})$$

$$s_B = 327 \text{ m} \qquad \textit{Ans.}$$

Velocity. To obtain the velocity of the rocket just before it hits the ground, we can apply Eq. 12–6 between points B and C, Fig. 12–4.

$$(+\uparrow) \qquad v_C^2 = v_B^2 + 2a_c(s_C - s_B)$$

$$= 0 + 2(-9.81 \text{ m/s}^2)(0 - 327 \text{ m})$$

$$v_C = -80.1 \text{ m/s} = 80.1 \text{ m/s} \downarrow \qquad \textit{Ans.}$$

The negative root was chosen since the rocket is moving downward.

Similarly, Eq. 12–6 may also be applied between points A and C, i.e.,

$$(+\uparrow) \qquad v_C^2 = v_A^2 + 2a_c(s_C - s_A)$$

$$= (75 \text{ m/s})^2 + 2(-9.81 \text{ m/s}^2)(0 - 40 \text{ m})$$

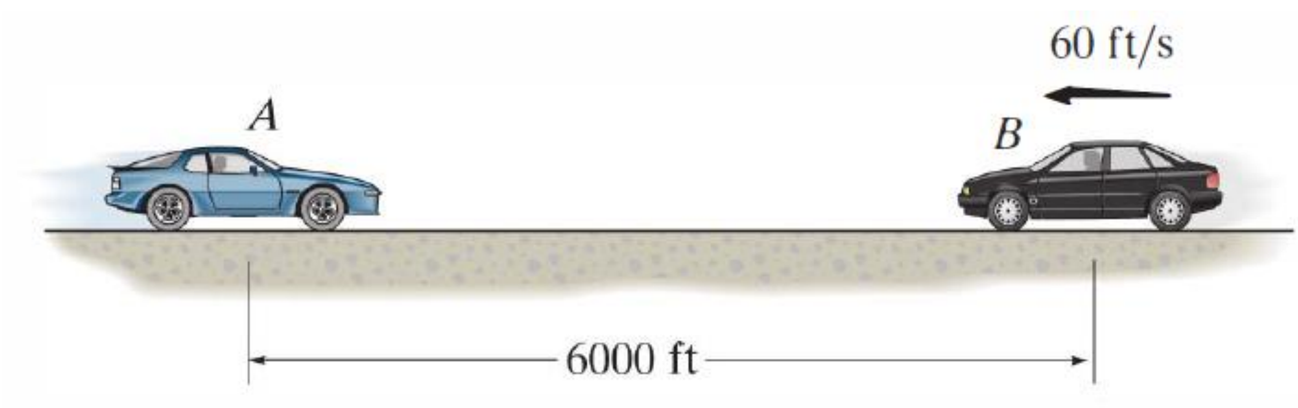
$$v_C = -80.1 \text{ m/s} = 80.1 \text{ m/s} \downarrow \qquad \textit{Ans.}$$

PROPLEMS:-

Q1/ Initially, the car travels along a straight road with a speed of 35 m/s. If the brakes are applied and the speed of the car is reduced to 10 m/s in 15 s, determine the constant deceleration of the car?

Q2/ A train starts from rest at a station and travels with a constant acceleration of 1 m/s^2. Determine the velocity of the train when (= 30 s and the distance traveled during this time?

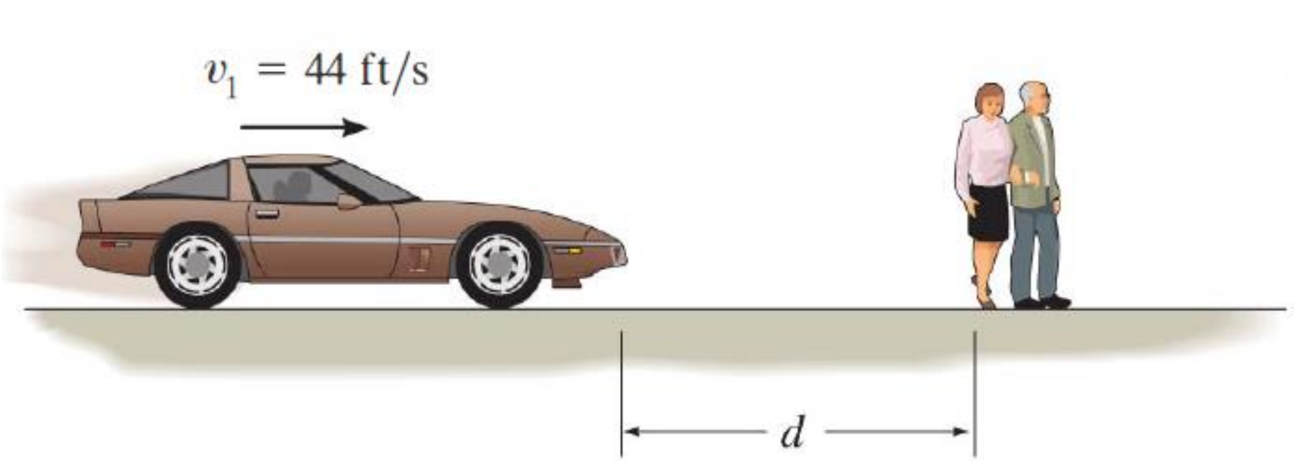
Q3/ Car A starts from rest at t = 0 and travels along a straight road with a constant acceleration of 6 ft/S^2 until it reaches a speed of 80 ft/s. Afterwards it maintains this speed. Also, when t = 0, car B located 6000 ft down the road is traveling towards A at a constant speed of 60 ft/s. Determine the distance traveled by car A when they pass each other.



Q4/ A particle travels along a straight line with a velocity $v = (12 - 3t^2)$ m/s, where t is in seconds. When t = 1 s, the particle is located 10 m to the left of the origin. Determine the acceleration when t = 4 s, the displacement from t = 0 to t = 10 s, and the distance the particle travels during this time period?

Q5/ The acceleration of a particle traveling along a straight line is a = k/v, where k is a constant. If S = 0, v = V_o when t= 0, determine the velocity of the particle as a function of time t.

Q6/ Tests reveal that a normal driver takes about 0.75 s before he or she can react to a situation to avoid a collision. It takes about 3 s for a driver having 0.1 % alcohol in his system to do the same. If such drivers are traveling on a straight road at 30 mph (44 ft/s) and their cars can decelerate at 2 ft/S^2, determine the shortest stopping distance d for each from the moment they see the pedestrians. Moral: If you must drink, please don't drive ?



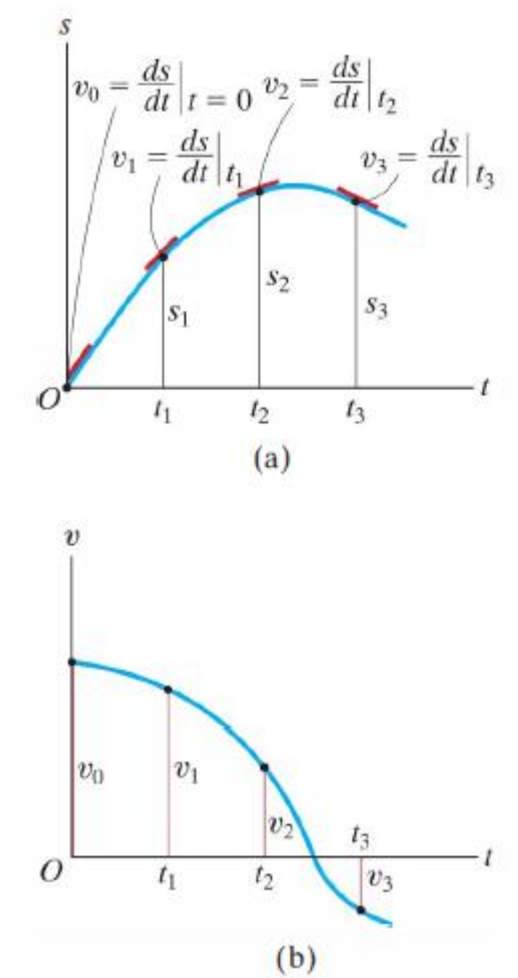
RECTILINEAR KINEMATICS: ERRATIC MOTION

When a particle has erratic or changing motion then its position, velocity, and acceleration cannot be described by a single continuous mathematical function along the entire path. Instead, a series of functions will be required to specify the motion at different intervals. For this reason, it is convenient to represent the motion as a graph. If a graph of the motion that relates any two of the variables s, v, a, t can be drawn, then this graph can be used to construct subsequent graphs relating two other variables Since d the variables are related by the differential relationships v = ds/d t, or a ds = v dv. Several situations occur frequently.

The s-t, v-t, and a-t Graphs. To construct the v-t graph given the s-t graph, Fig. , the equation v = ds/dt should be used, since it relates the variables s and t to v. This equation states that

$$\frac{ds}{dt} = v$$

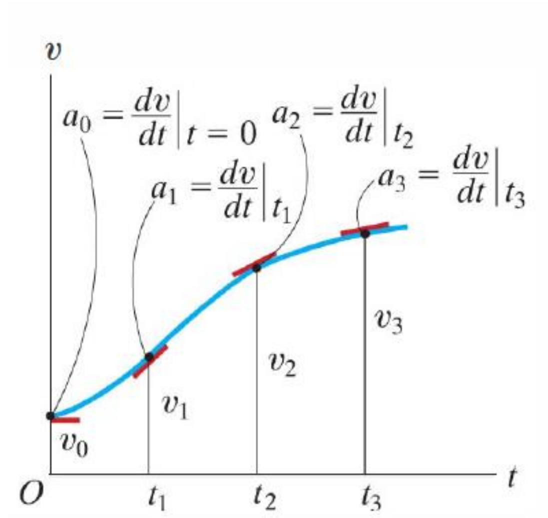
slope of s–t graph = velocity



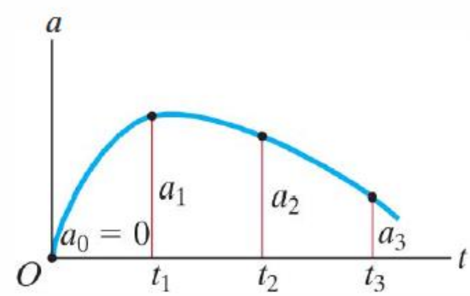
For example, by measuring the slope on the s-t graph when t = t_1, the velocity is v_1 , which is plotted in Fig.. The v-t graph can be constructed by plotting this and other values at each instant. The a-t graph can be constructed from the v-t graph in a similar manner, Fig., since

$$\frac{dv}{dt} = a$$

slope of v–t graph = acceleration



(a)



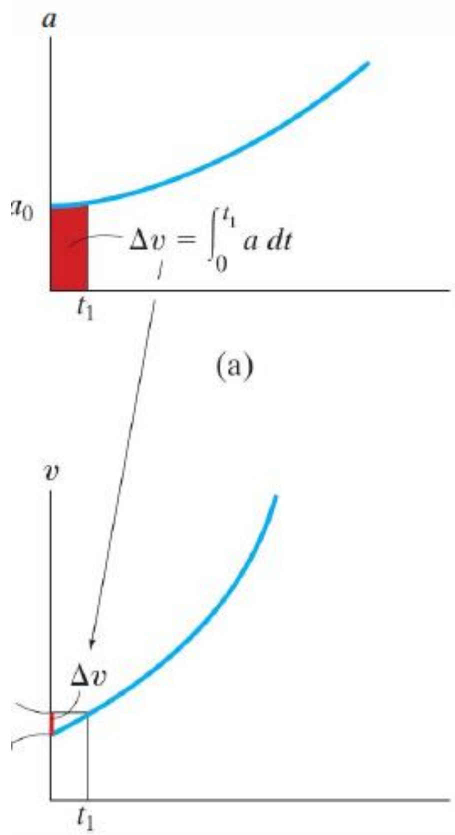
(b)

If the s–t curve for each interval of motion can be expressed by a mathematical function $s = s(t)$, then the equation of the v–t graph for the same interval can be obtained by differentiating this function with respect to time since $v = ds/dt$. Likewise, the equation of the a–t graph for the same interval can be determined by differentiating $v = v(t)$ since $a = dv/dt$. Since differentiation reduces a polynomial of degree n to that of degree $n - 1$, then if the s–t graph is parabolic (a second-degree curve), the v–t graph will be a sloping line (a first-degree curve), and the a–t graph will be a constant or a horizontal line (a zero-degree curve).

If the a-t graph is given, Fig. a, the v-t graph may be constructed using a = dv/dt, written as

$$\Delta v = \int a\, dt$$

change in velocity = area under a–t graph



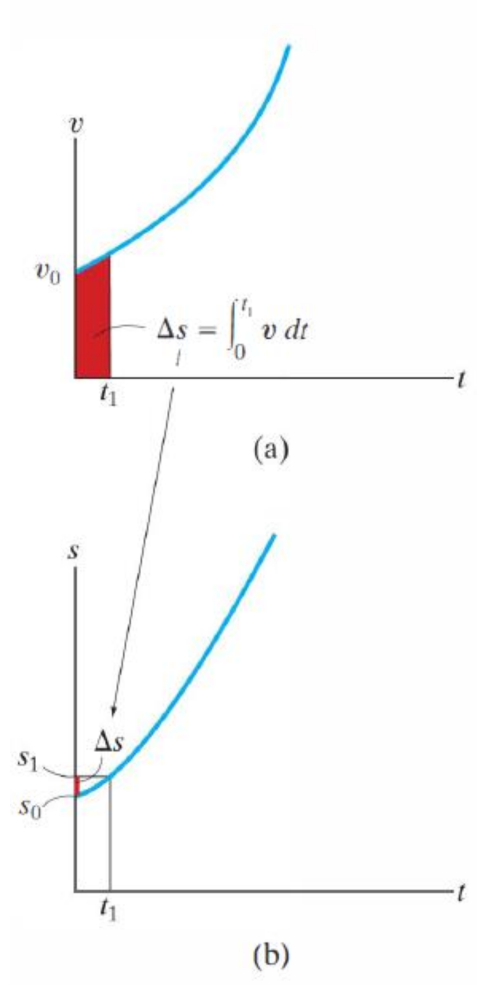
(a)

(b)

Hence, to construct the v–t graph, we begin with the particle's initial velocity v_0 and then add to this small increments of area (Δv) determined from the a–t graph. In this manner successive points, $v_1 = v_0 + \Delta v$, etc., for the v–t graph are determined, Fig. 12–9b. Notice that an algebraic addition of the area increments of the a–t graph is necessary, since areas lying above the t axis correspond to an increase in v ("positive" area), whereas those lying below the axis indicate a decrease in v ("negative" area).

In the same manner as stated above, we begin with the particle's initial position S_o and add (algebraically) to this small area increments Δs determined from the v-t graph

If segments of the a-t graph can be described by a series of equations, then each of these equations can be integrated to yield equations describing the corresponding segments of the v-t graph. In a similar manner, the s-t graph can be obtained by integrating the equations which describe the segments of the v-t graph. As a result, if the a-t graph is linear (a first-degree curve), integration will yield a v-t graph that is parabolic (a second-degree curve) and an s-t graph that is cubic (third-degree curve).



The v-s and a-s Graphs.

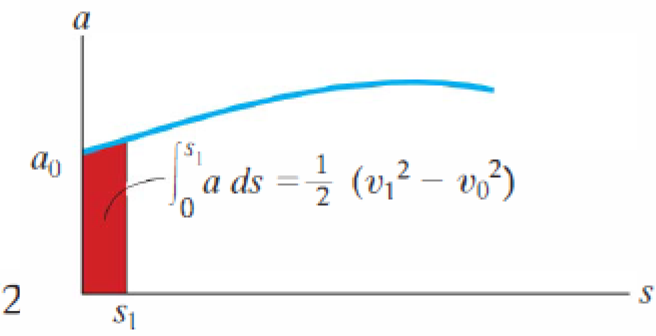
If the a-s graph can be constructed, then points on the v-s graph can be determined by using v dv = a ds. Integrating this equation between the limits v = Vo at s = So and v = V1 at s = s1, we have,

$$\frac{1}{2}(v_1^2 - v_0^2) = \underbrace{\int_{s_0}^{s_1} a\,ds}_{\text{area under } a\text{–}s \text{ graph}}$$

Therefore, if the red area in Fig. a is determined, and the initial

velocity Vo at So = 0 is known, then

$$v_1 = \left(2\int_{s_0}^{s_1} a\,ds + v_0^2\right)^{1/2}$$

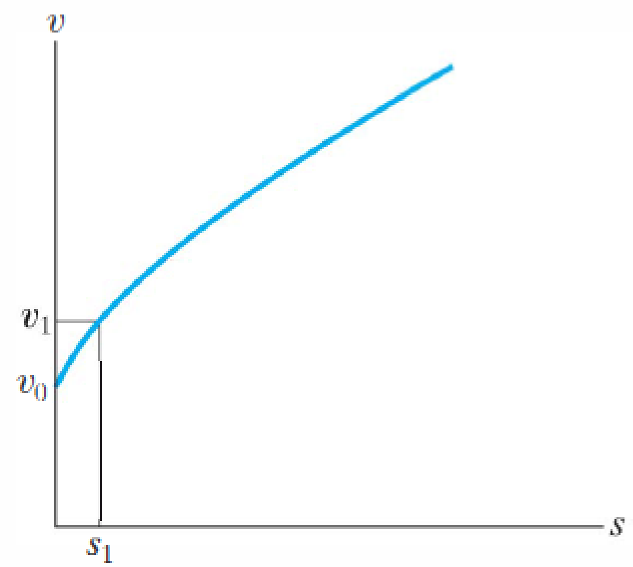


(a)

Fig. b.Successive points on the v-s graph can be constructed in this manner. If the v-s graph is known, the acceleration a at any position s can be determined using a ds = v dv, written as

$$a = v\left(\frac{dv}{ds}\right)$$

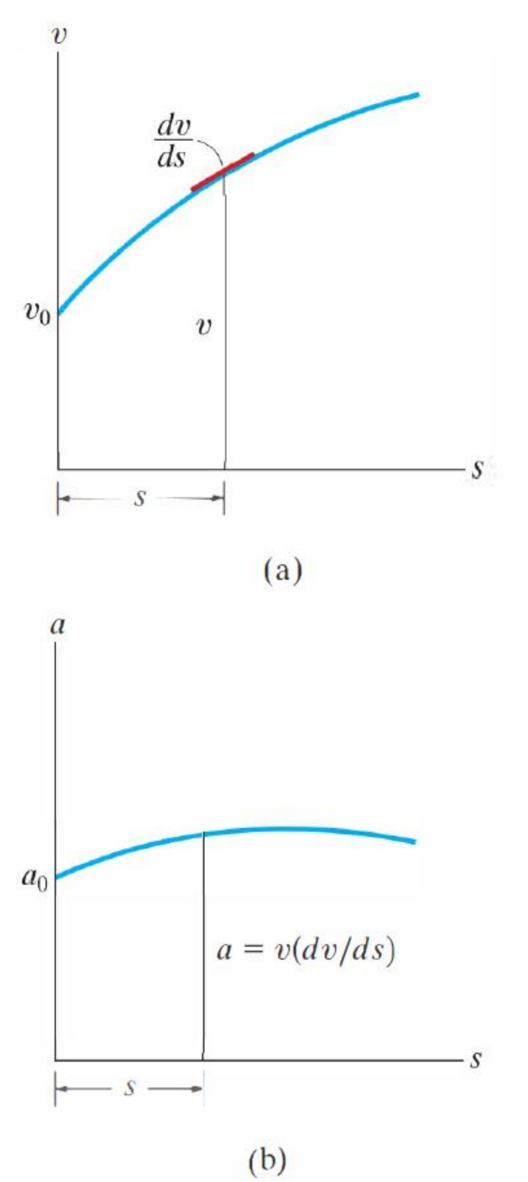
acceleration = velocity times slope of v–s graph



(b)

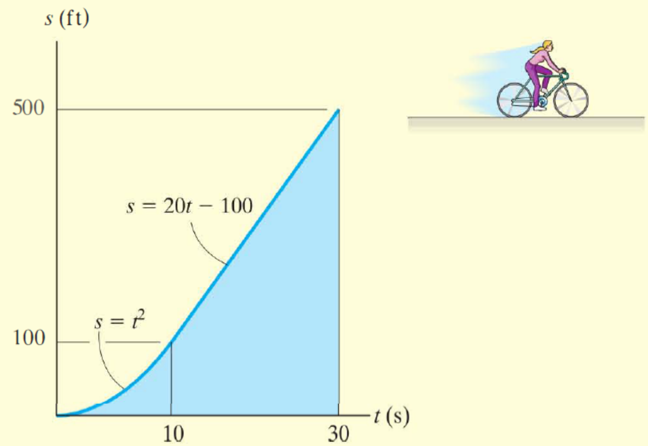
Thus, at any point (s, v) in Fig. a, the slope dv/ds of the v-s graph is measured. Then with **v** and dv/ds known, the value of a can be calculated, Fig b. The v-s graph can also be constructed from the a-s graph, or vice versa, by approximating the known graph in various intervals with mathematical functions, v = f(s) or a = g(s), and then using

a ds = v dv to obtain the other graph.



EXA M P L E -1-

A bicycle moves along a straight road such that its position is described by the graph shown in Fig. 12–13*a*. Construct the v–t and a–t graphs for $0 \le t \le 30$ s.



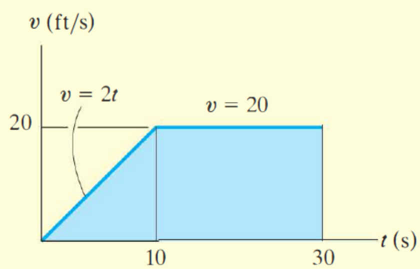
(a)

SOLUTION

v–t Graph. Since $v = ds/dt$, the v–t graph can be determined by differentiating the equations defining the s–t graph, Fig. 12–13*a*. We have

$$0 \le t < 10 \text{ s}; \qquad s = (t^2) \text{ ft} \qquad v = \frac{ds}{dt} = (2t) \text{ ft/s}$$

$$10 \text{ s} < t \le 30 \text{ s}; \qquad s = (20t - 100) \text{ ft} \qquad v = \frac{ds}{dt} = 20 \text{ ft/s}$$



(b)

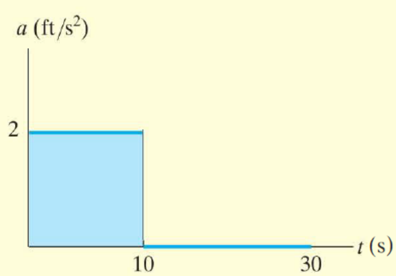
The results are plotted in Fig. 12–13*b*. We can also obtain specific values of v by measuring the *slope* of the s–t graph at a given instant. For example, at $t = 20$ s, the slope of the s–t graph is determined from the straight line from 10 s to 30 s, i.e.,

$$t = 20 \text{ s}; \qquad v = \frac{\Delta s}{\Delta t} = \frac{500 \text{ ft} - 100 \text{ ft}}{30 \text{ s} - 10 \text{ s}} = 20 \text{ ft/s}$$

a–t Graph. Since $a = dv/dt$, the a–t graph can be determined by differentiating the equations defining the lines of the v–t graph. This yields

$$0 \le t < 10 \text{ s}; \qquad v = (2t) \text{ ft/s} \qquad a = \frac{dv}{dt} = 2 \text{ ft/s}^2$$

$$10 < t \le 30 \text{ s}; \qquad v = 20 \text{ ft/s} \qquad a = \frac{dv}{dt} = 0$$



(c)

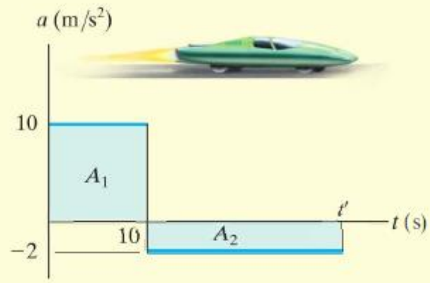
Fig. 12–13

The results are plotted in Fig. 12–13*c*.

NOTE: Show that $a = 2 \text{ ft/s}^2$ when $t = 5$ s by measuring the slope of the v–t graph.

EXA M P L E -2-

The car in Fig. 12–14*a* starts from rest and travels along a straight track such that it accelerates at $10\ m/s^2$ for 10 s, and then decelerates at $2\ m/s^2$. Draw the v–t and s–t graphs and determine the time t' needed to stop the car. How far has the car traveled?



(a)

SOLUTION

v–t Graph. Since $dv = a\,dt$, the v–t graph is determined by integrating the straight-line segments of the a–t graph. Using the *initial condition* $v = 0$ when $t = 0$, we have

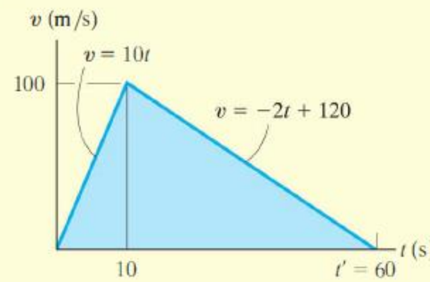
$$0 \le t < 10\text{ s};\qquad a = (10)\ \text{m/s}^2;\quad \int_0^v dv = \int_0^t 10\,dt,\qquad v = 10t$$

When $t = 10$ s, $v = 10(10) = 100$ m/s. Using this as the *initial condition* for the next time period, we have

$$10\text{ s} < t \le t';\ a = (-2)\ \text{m/s}^2;\ \int_{100\text{ m/s}}^{v} dv = \int_{10\text{ s}}^{t} -2\,dt,\ v = (-2t + 120)\ \text{m/s}$$

When $t = t'$ we require $v = 0$. This yields, Fig. 12–14*b*,

$$t' = 60\text{ s}\qquad \textit{Ans.}$$



(b)

A more direct solution for t' is possible by realizing that the area under the a–t graph is equal to the change in the car's velocity. We require $\Delta v = 0 = A_1 + A_2$, Fig. 12–14*a*. Thus

$$0 = 10\ \text{m/s}^2(10\text{ s}) + (-2\ \text{m/s}^2)(t' - 10\text{ s})$$

$$t' = 60\text{ s}\qquad \textit{Ans.}$$

s–t Graph. Since $ds = v\,dt$, integrating the equations of the v–t graph yields the corresponding equations of the s–t graph. Using the *initial condition* $s = 0$ when $t = 0$, we have

$$0 \le t \le 10\text{ s};\qquad v = (10t)\ \text{m/s};\quad \int_0^s ds = \int_0^t 10t\,dt,\qquad s = (5t^2)\ \text{m}$$

When $t = 10$ s, $s = 5(10)^2 = 500$ m. Using this *initial condition*,

$$10\text{ s} \le t \le 60\text{ s};\ v = (-2t + 120)\ \text{m/s};\ \int_{500\text{ m}}^{s} ds = \int_{10\text{ s}}^{t} (-2t + 120)\,dt$$

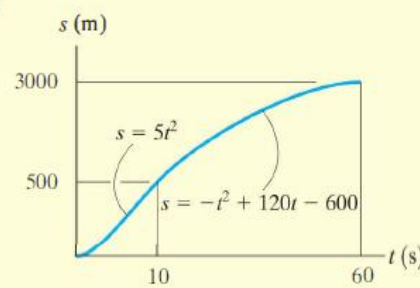
$$s - 500 = -t^2 + 120t - [-(10)^2 + 120(10)]$$

$$s = (-t^2 + 120t - 600)\ \text{m}$$

When $t' = 60$ s, the position is

$$s = -(60)^2 + 120(60) - 600 = 3000\text{ m}\qquad \textit{Ans.}$$

The s–t graph is shown in Fig. 12–14*c*.



(c)

Fig. 12–14

NOTE: A direct solution for s is possible when $t' = 60$ s, since the *triangular area* under the v–t graph would yield the displacement $\Delta s = s - 0$ from $t = 0$ to $t' = 60$ s. Hence,

$$\Delta s = \tfrac{1}{2}(60\text{ s})(100\text{ m/s}) = 3000\text{ m}\qquad \textit{Ans.}$$

EXA M P L E -3-

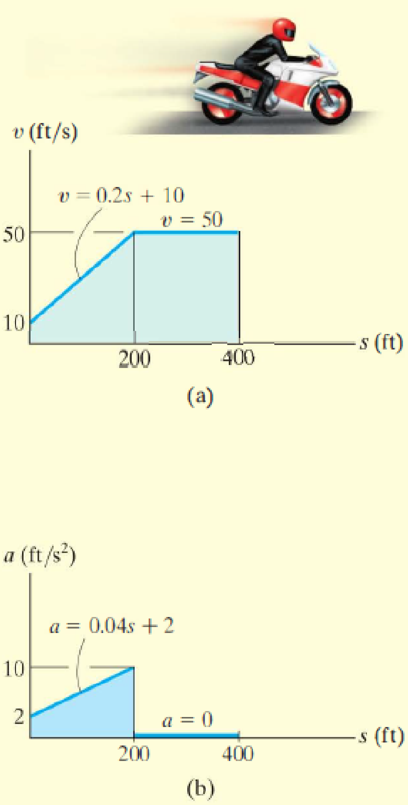


Fig. 12–15

The v–s graph describing the motion of a motorcycle is shown in Fig. 12–15*a*. Construct the a–s graph of the motion and determine the time needed for the motorcycle to reach the position $s = 400$ ft.

SOLUTION

a–s Graph. Since the equations for segments of the v–s graph are given, the a–s graph can be determined using $a\,ds = v\,dv$.

$0 \le s < 200$ ft; $\quad v = (0.2s + 10)$ ft/s

$$a = v\frac{dv}{ds} = (0.2s + 10)\frac{d}{ds}(0.2s + 10) = 0.04s + 2$$

$200 \text{ ft} < s \le 400$ ft; $\quad v = 50$ ft/s

$$a = v\frac{dv}{ds} = (50)\frac{d}{ds}(50) = 0$$

The results are plotted in Fig. 12–15*b*.

Time. The time can be obtained using the v–s graph and $v = ds/dt$, because this equation relates v, s, and t. For the first segment of motion, $s = 0$ when $t = 0$, so

$0 \le s < 200$ ft; $\quad v = (0.2s + 10)$ ft/s; $\quad dt = \dfrac{ds}{v} = \dfrac{ds}{0.2s + 10}$

$$\int_0^t dt = \int_0^s \frac{ds}{0.2s + 10}$$

$$t = (5\ln(0.2s + 10) - 5\ln 10)\text{ s}$$

At $s = 200$ ft, $t = 5\ln[0.2(200) + 10] - 5\ln 10 = 8.05$ s. Therefore, using these initial conditions for the second segment of motion,

$200 \text{ ft} < s \le 400$ ft; $\quad v = 50$ ft/s; $\quad dt = \dfrac{ds}{v} = \dfrac{ds}{50}$

$$\int_{8.05\text{ s}}^t dt = \int_{200\text{ m}}^s \frac{ds}{50};$$

$$t - 8.05 = \frac{s}{50} - 4; \quad t = \left(\frac{s}{50} + 4.05\right)\text{s}$$

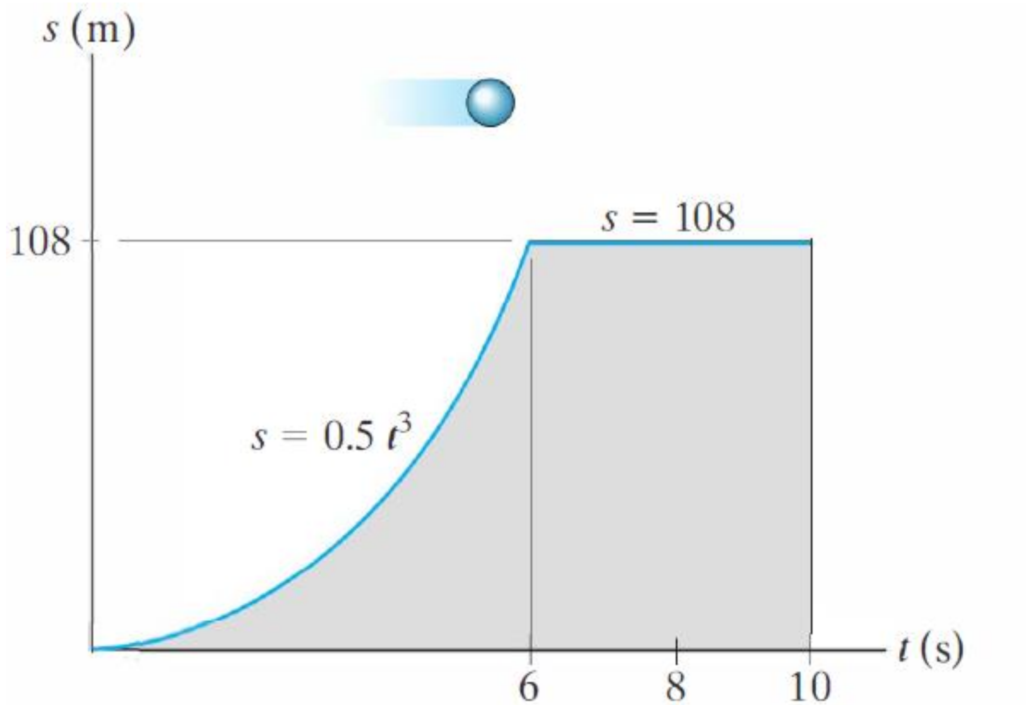
Therefore, at $s = 400$ft,

$$t = \frac{400}{50} + 4.05 = 12.0\text{ s} \qquad \textit{Ans.}$$

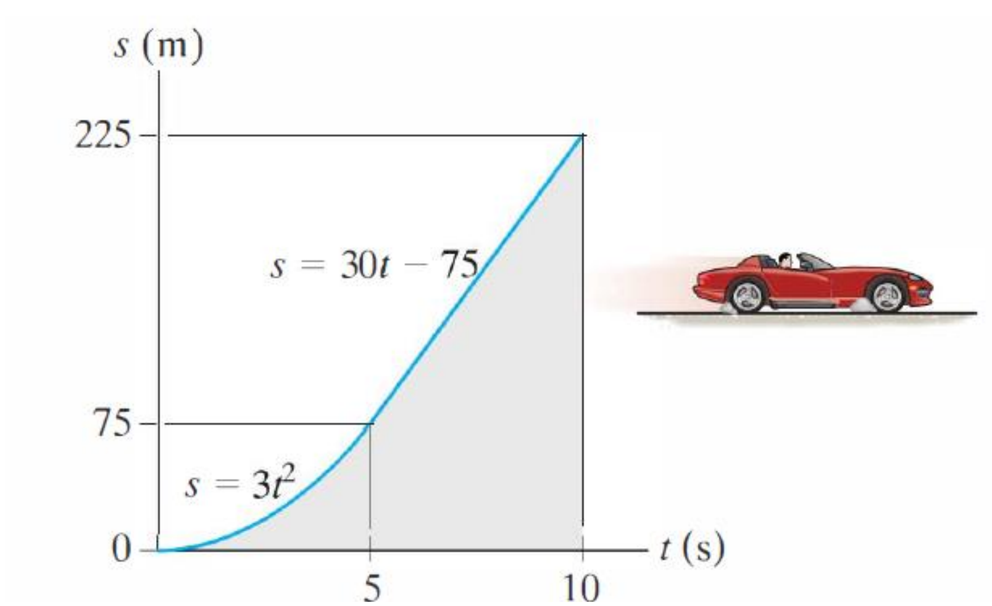
NOTE: The graphical results can be checked in part by calculating slopes. For example, at $s = 0$, $a = v(dv/ds) = 10(50 - 10)/200 = 2$ m/s^2. Also, the results can be checked in part by inspection. The v–s graph indicates the initial increase in velocity (acceleration) followed by constant velocity ($a = 0$).

PROPLEMS:-

Q1/ The particle travels along a straight track such that its position is described by the s-t graph. Construct the v-t graph for the same time interval.

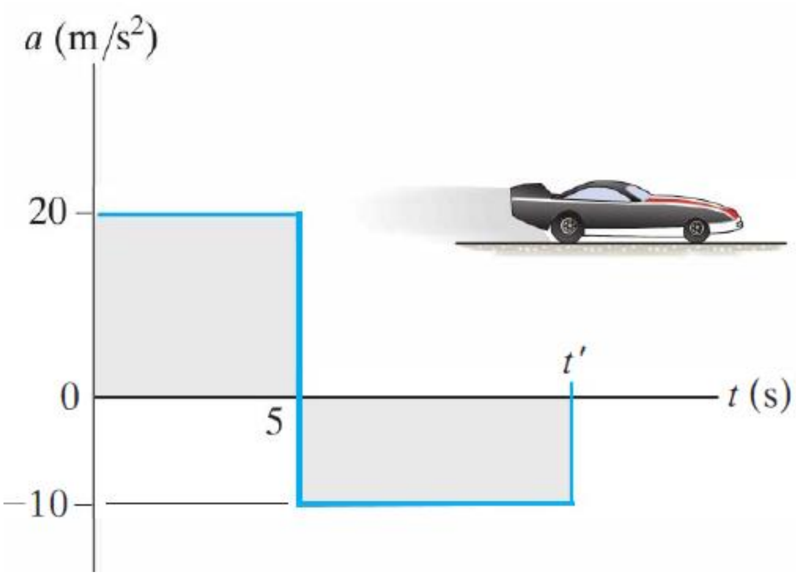


Q2/ The sports car travels along a straight road such that its position is described by the graph. Construct the v-t and a-t graphs for the time interval **0 ≤ t ≥10 s.**

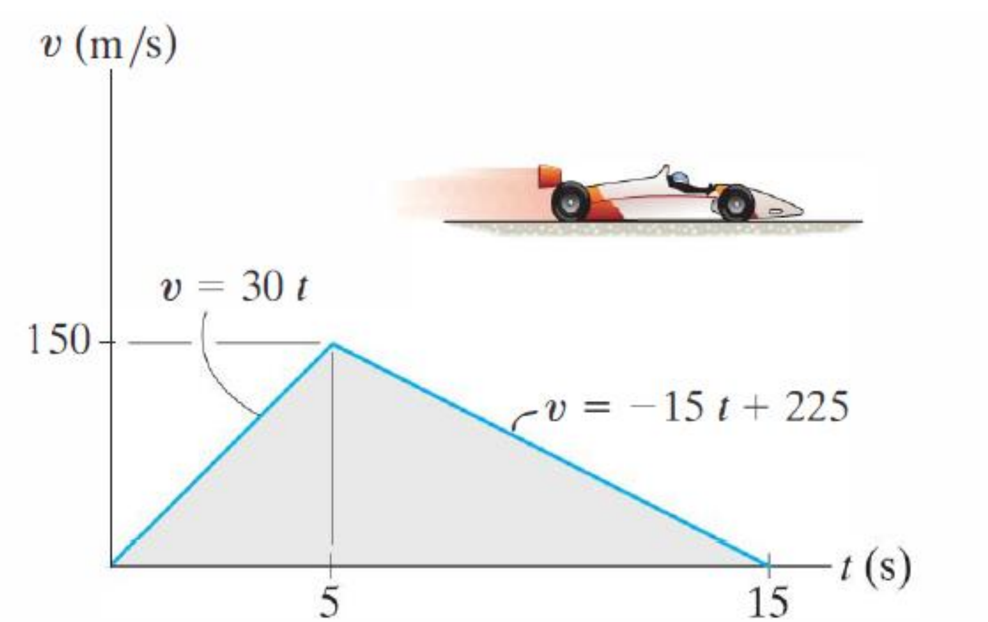


Q3/ The

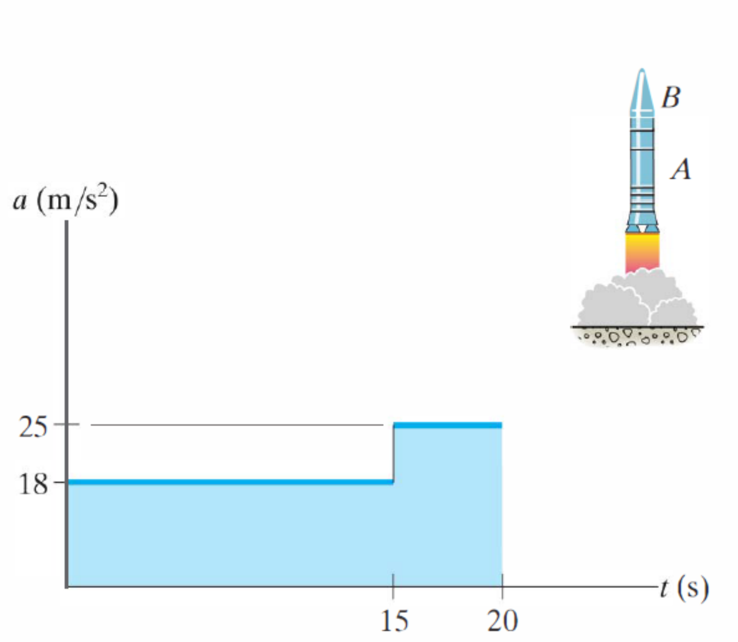
dragster starts from rest and has an acceleration described by the graph. Construct the v-t graph for the time interval **$0 \le t \ge$** t' **s.**, where t' is the time for the car to come to rest.



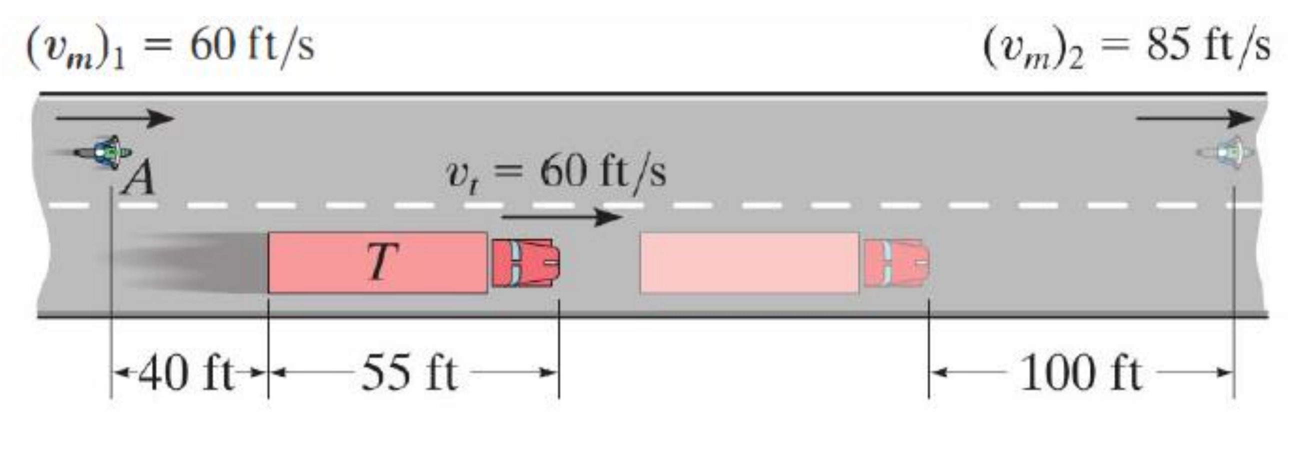
Q4/ The dragster start from rest and has a velocity described by the graph. Construct the s-t graph during the time interval **$0 \le t \ge$** 15s. Also, determine the total distance traveled during this time interval.



Q5/ A two-stage missile is fired vertically from rest with the acceleration shown. In 15 s the first stage A burns out and the second stage B ignites. Plot the v-t and s-t graphs which describe the two-stage motion of the missile for $0 \leq t \geq 20$ s.



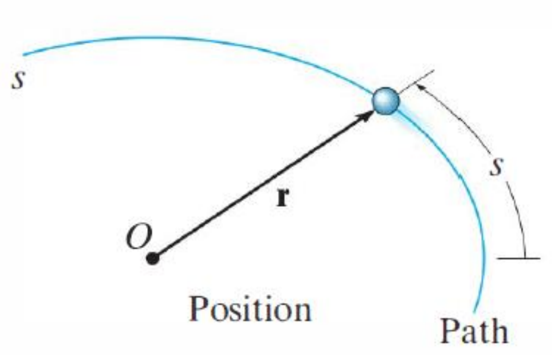
Q6/ A motorcyclist at A is traveling at 60 ft/s when he wishes to pass the truck T which is traveling at a constant speed of 60 ft/s. To do so the motorcyclist accelerates at 6 ft/S2 until reaching a maximum speed of 85 ft/s. If he then maintains this speed, determine the time needed for him to reach a point located 100 ft in front of the truck. Draw the v-t and s-t graphs for the motorcycle during this time.



General Curvilinear Motion

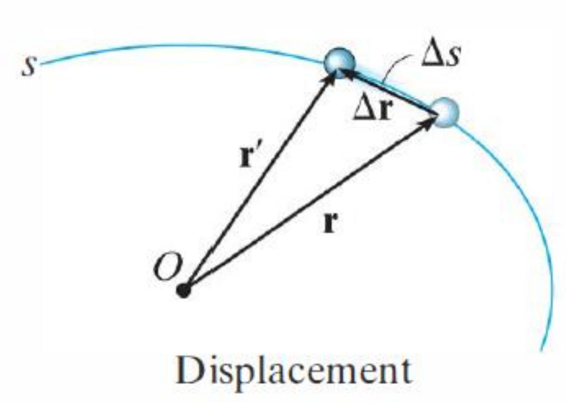
Curvilinear motion occurs when a particle moves along a curved path. Since this path is often described in three dimensions, vector analysis will be used to formulate the particle's position, velocity, and acceleration.* In this section the general aspects of curvilinear motion are discussed, and in subsequent sections we will consider three types of coordinate systems often used to analyze this motion.

Position. Consider a particle located at a point on a space curve defined by the path function s(t) Fig a. The position of the particle, measured from a fixed point **O**, will be designated by the position vector (r= r(t)) Notice that both the magnitude and direction of this vector will change as the particle moves along the curve.



(a)

Displacement. Suppose that during a small time interval Δt the particle moves a distance Δs along the curve to a new position, defined by r' = r + Δr, Fig b. The displacement Δr represents the change in the particle's position and is determined by vector subtraction; i.e., Δr = r' - r.



(b)

Velocity. During the time Δt the average velocity of the particle is

$$\mathbf{v}_{avg} = \frac{\Delta \mathbf{r}}{\Delta t}$$

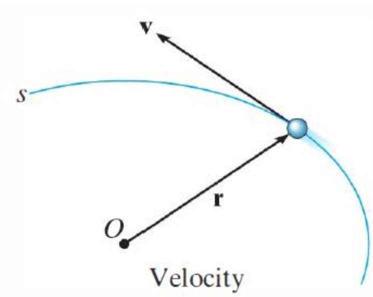
The *instantaneous velocity* is determined from this equation by letting $\Delta t \to 0$, and consequently the direction of $\Delta\mathbf{r}$ *approaches* the *tangent* to the curve. Hence, $\mathbf{v} = \lim_{\Delta t \to 0} (\Delta\mathbf{r}/\Delta t)$ or

$$\mathbf{v} = \frac{d\mathbf{r}}{dt}$$

Since $d\mathbf{r}$ will be tangent to the curve, the *direction* of $\mathbf{v}$ is also *tangent to the curve*, Fig. 12–16*c*. The *magnitude* of $\mathbf{v}$, which is called the *speed*, is obtained by realizing that the length of the straight line segment $\Delta\mathbf{r}$ in Fig. 12–16*b* approaches the arc length Δs as $\Delta t \to 0$, we have $v = \lim_{\Delta t \to 0} (\Delta r/\Delta t) = \lim_{\Delta t \to 0} (\Delta s/\Delta t)$, or

Thus, the speed can be obtained by differentiating the path function s with respect to time.

$$v = \frac{ds}{dt}$$



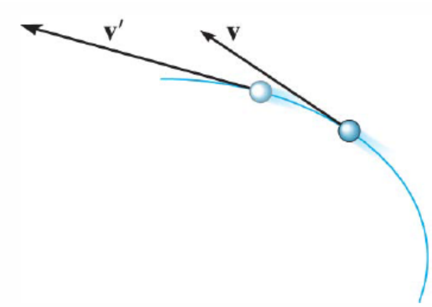
Velocity
(c)

Acceleration. **If the particle has a velocity v at time t and a velocity v' = v + Δv at t + Δt, Fig. d, then the average acceleration of the particle during the time interval**

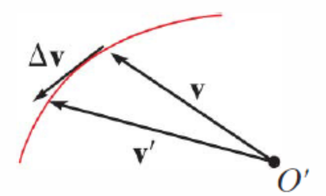
$$\mathbf{a}_{avg} = \frac{\Delta \mathbf{v}}{\Delta t}$$

Δt is

Where Δ v = v' - v. To study this time rate of change, the two velocity vectors in Fig. d are plotted in Fig. e such that their tails are located at the fixed point 0' and their arrowheads touch points on a curve. This curve is called a hodograph, and when constructed, it describes the locus of points for the arrowhead of the velocity vector in the same manner as the path s describes the locus of points for the arrowhead of the position vector, Fig. a.



(d)



(e)

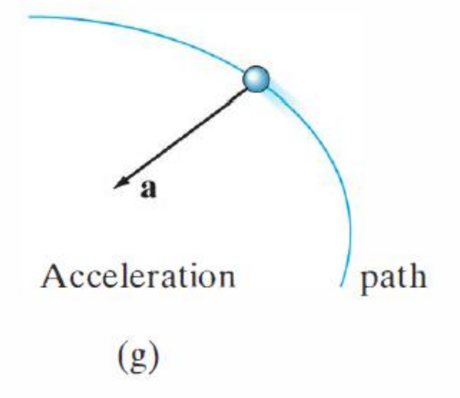
To obtain the instantaneous acceleration, let Δt → **0** in the above equation. In the limit Δ v will approach the tangent to the hodograph, and so

$$\mathbf{a} = \lim_{\Delta t \to 0} (\Delta \mathbf{v}/\Delta t), \text{ or}$$

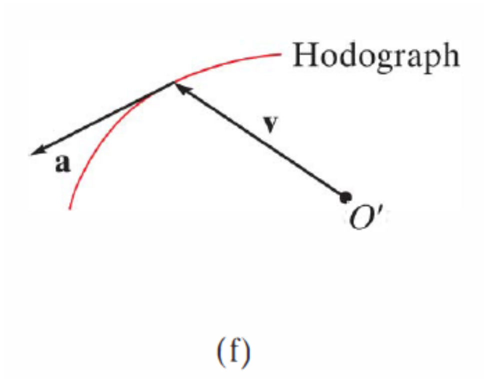
$$\mathbf{a} = \frac{d\mathbf{v}}{dt}$$

We can also write as $$\mathbf{a} = \frac{d^2\mathbf{r}}{dt^2}$$

By definition of the derivative, a acts tangent to the hodograph, Fig. f, and, in general it is not tangent to the path oj motion, Fig. g. To clarify this point, realize that Δv and consequently a must account for the change made in both the magnitude and direction of the velocity v as the particle moves from one point to the next along the path, Fig. d . However, in order for the particle to follow any curved path, the directional change always "swings" the velocity vector toward the "inside" or "concave side" of the path, and therefore a cannot remain tangent to the path. In summary, v is always tangent to the path and a is always tangent to the hodograph.



(g)



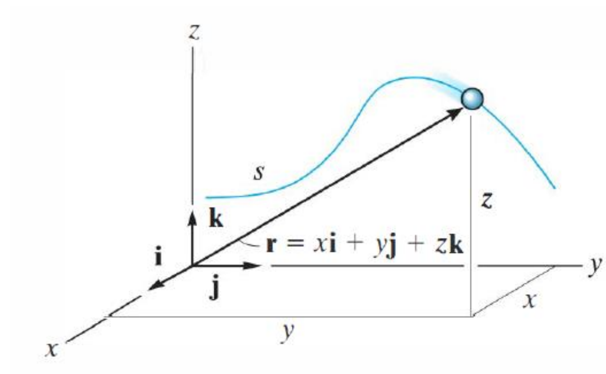
(f)

Curvilinear Motion: Rectangular Components

Occasionally the motion of a particle can best be described along a path that can be expressed in terms of its x, y, z coordinates.

Position. If the particle is at point (x, y, z) on the curved path s shown in Fig a, then its location is defined by the position vector

$$\mathbf{r} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$$



Position
(a)

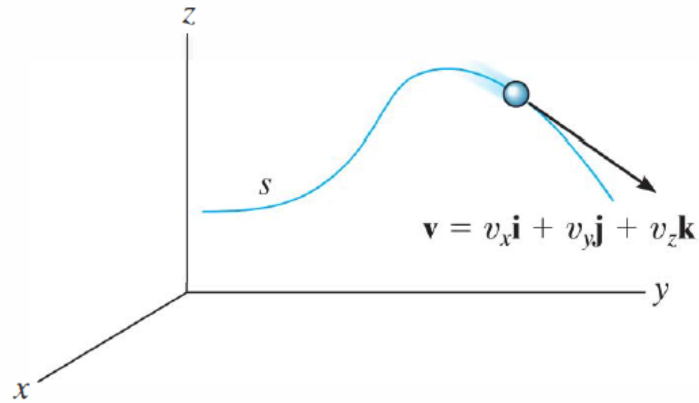
When the particle moves, the x, y, z components of r will be functions of time; i.e., X = x(t), y = y(t), z = z(t), so that r = (t).At any instant the magnitude of r is defined by the Eq.

$$r = \sqrt{x^2 + y^2 + z^2}$$

And the direction of r is specified by the unit vector $u_r = \mathbf{r}/r$.

Velocity. The first time derivative of r yields the velocity of the particle. Hence,

$$\mathbf{v} = \frac{d\mathbf{r}}{dt} = \frac{d}{dt}(x\mathbf{i}) + \frac{d}{dt}(y\mathbf{j}) + \frac{d}{dt}(z\mathbf{k})$$



Velocity

(b)

When taking this derivative, it is necessary to account for changes in both the magnitude and direction of each of the vector's components. For example, the derivative of the i component of r is

$$\frac{d}{dt}(x\mathbf{i}) = \frac{dx}{dt}\mathbf{i} + x\frac{d\mathbf{i}}{dt}$$

The second term on the right side is zero, provided the x, y, z reference frame is fixed, and therefore the direction (and the magnitude) of i does not change with time. Differentiation of the j and k components may be carried out in a similar manner, which yields the final result,

$$\mathbf{v} = \frac{d\mathbf{r}}{dt} = v_x\mathbf{i} + v_y\mathbf{j} + v_z\mathbf{k}$$

Where

$$v_x = \dot{x} \quad v_y = \dot{y} \quad v_z = \dot{z}$$

The "dot" notation X, y, z represents the first time derivatives of x = x(t), y = y(t), z = z(t), respectively.The velocity has a magnitude that is found from

$$v = \sqrt{v_x^2 + v_y^2 + v_z^2}$$

and a direction that is specified by the unit vector $\mathbf{u}_v$ = v/υ. this direction is always tangent to the path, as shown in Fig.b.

Acceleration. The acceleration of the particle is obtained by taking the first time derivative and we have

$$\mathbf{a} = \frac{d\mathbf{v}}{dt} = a_x\mathbf{i} + a_y\mathbf{j} + a_z\mathbf{k}$$

Where

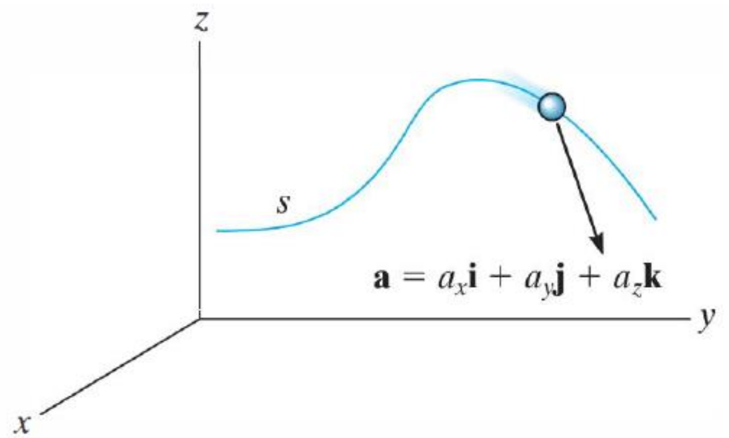
$$a_x = \dot{v}_x = \ddot{x}$$
$$a_y = \dot{v}_y = \ddot{y}$$
$$a_z = \dot{v}_z = \ddot{z}$$

Here a_x, a_y, a_z represent, respectively, the first time derivatives of $v_x = v_x(t)$, $v_y = v_y(t)$, $v_z = v_z(t)$, or the second time derivatives of the functions $x = x(t)$, $y = y(t)$, $z = z(t)$.

The acceleration has a *magnitude*

$$a = \sqrt{a_x^2 + a_y^2 + a_z^2}$$

and a direction specified by the unit vector $U_a = \mathbf{a}/a$, Since a represents the time rate of change in both the magnitude and direction of the velocity, in general a will not be tangent to the path, Fig. C.



Acceleration

(c)

Procedure of analysis

Coordinate System.

- A rectangular coordinate system can be used to solve problems for which the motion can conveniently be expressed in terms of its x, y, z components.

Kinematic Quantities.

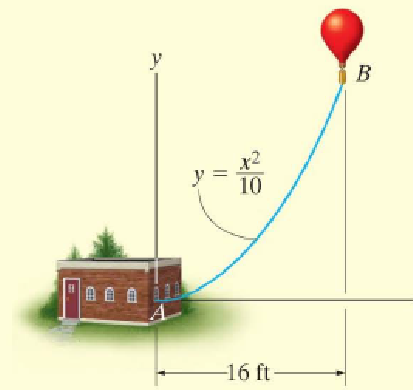
- Since *rectilinear motion* occurs along *each coordinate axis*, the motion along each axis is found using $v = ds/dt$ and $a = dv/dt$; or in cases where the motion is not expressed as a function of time, the equation $a\,ds = v\,dv$ can be used.

- In two dimensions, the equation of the path $y = f(x)$ can be used to relate the x and y components of velocity and acceleration by applying the chain rule of calculus. A review of this concept is given in Appendix C.

- Once the x, y, z components of $\mathbf{v}$ and $\mathbf{a}$ have been determined, the magnitudes of these vectors are found from the Pythagorean theorem, Eq. B–3, and their coordinate direction angles from the components of their unit vectors, Eqs. B–4 and B–5.

EXA M P L E-1-

At any instant the horizontal position of the weather balloon in Fig. 12–18*a* is defined by $x = (8t)$ ft, where t is in seconds. If the equation of the path is $y = x^2/10$, determine the magnitude and direction of the velocity and the acceleration when $t = 2$ s.



(a)

SOLUTION

Velocity. The velocity component in the x direction is

$$v_x = \dot{x} = \frac{d}{dt}(8t) = 8 \text{ ft/s} \rightarrow$$

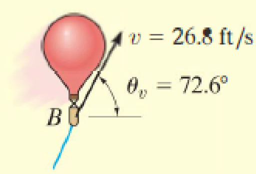
To find the relationship between the velocity components we will use the chain rule of calculus. (See Appendix A for a full explanation.)

$$v_y = \dot{y} = \frac{d}{dt}(x^2/10) = 2x\dot{x}/10 = 2(16)(8)/10 = 25.6 \text{ ft/s} \uparrow$$

When $t = 2$ s, the magnitude of velocity is therefore

$$v = \sqrt{(8 \text{ ft/s})^2 + (25.6 \text{ ft/s})^2} = 26.8 \text{ ft/s} \qquad \textit{Ans.}$$

The direction is tangent to the path, Fig. 12–18*b*, where



(b)

$$\theta_v = \tan^{-1}\frac{v_y}{v_x} = \tan^{-1}\frac{25.6}{8} = 72.6° \qquad \textit{Ans.}$$

Acceleration. The relationship between the acceleration components is determined using the chain rule. (See Appendix C.) We have

$$a_x = \dot{v}_x = \frac{d}{dt}(8) = 0$$

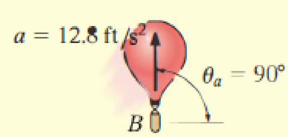
$$a_y = \dot{v}_y = \frac{d}{dt}(2x\dot{x}/10) = 2(\dot{x})\dot{x}/10 + 2x(\ddot{x})/10$$

$$= 2(8)^2/10 + 2(16)(0)/10 = 12.8 \text{ ft/s}^2 \uparrow$$

Thus,

$$a = \sqrt{(0)^2 + (12.8)^2} = 12.8 \text{ ft/s}^2 \qquad \textit{Ans.}$$

The direction of **a**, as shown in Fig. 12–18*c*, is



(c)

$$\theta_a = \tan^{-1}\frac{12.8}{0} = 90° \qquad \textit{Ans.}$$

Fig. 12–18

EXA M P L E-2-

For a short time, the path of the plane in Fig. 12–19*a* is described by $y = (0.001x^2)$ m. If the plane is rising with a constant velocity of 10 m/s, determine the magnitudes of the velocity and acceleration of the plane when it is at $y = 100$ m.

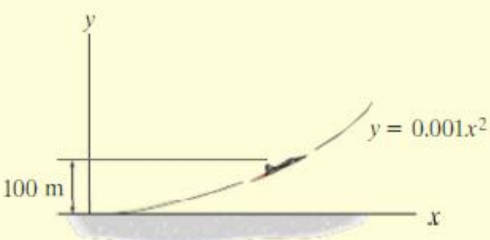
SOLUTION

When $y = 100$ m, then $100 = 0.001x^2$ or $x = 316.2$ m. Also, since $v_y = 10$ m/s, then

$$y = v_y t; \qquad 100\text{ m} = (10\text{ m/s})\,t \qquad t = 10\text{ s}$$

Velocity. Using the chain rule (see Appendix C) to find the relationship between the velocity components, we have

$$v_y = \dot{y} = \frac{d}{dt}(0.001x^2) = (0.002x)\dot{x} = 0.002xv_x \qquad (1)$$



(a)

Thus

$$10\text{ m/s} = 0.002(316.2\text{ m})(v_x)$$
$$v_x = 15.81\text{ m/s}$$

The magnitude of the velocity is therefore

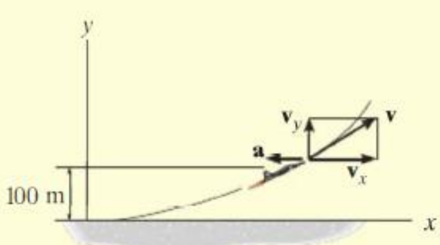
$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{(15.81\text{ m/s})^2 + (10\text{ m/s})^2} = 18.7\text{ m/s} \qquad \textit{Ans.}$$

Acceleration. Using the chain rule, the time derivative of Eq. (1) gives the relation between the acceleration components.

$$a_y = \dot{v}_y = 0.002\dot{x}v_x + 0.002x\dot{v}_x = 0.002(v_x^2 + xa_x)$$

When $x = 316.2$ m, $v_x = 15.81$ m/s, $\dot{v}_y = a_y = 0$,

$$0 = 0.002((15.81\text{ m/s})^2 + 316.2\text{ m}(a_x))$$
$$a_x = -0.791\text{ m/s}^2$$



(b)

Fig. 12–19

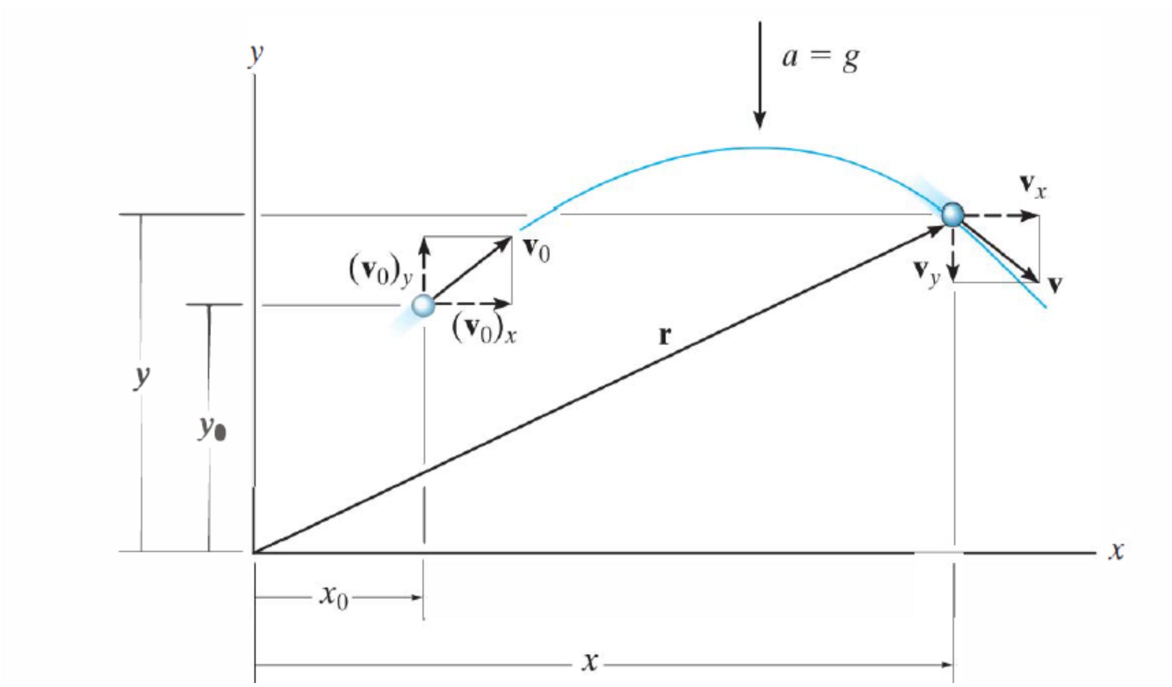
The magnitude of the plane's acceleration is therefore

$$a = \sqrt{a_x^2 + a_y^2} = \sqrt{(-0.791\text{ m/s}^2)^2 + (0\text{ m/s}^2)^2}$$
$$= 0.791\text{ m/s}^2 \qquad \textit{Ans.}$$

Motion of a Projectile

The free-flight motion of a projectile is often studied in terms of its rectangular components. To illustrate the kinematic analysis, consider a projectile launched at point (x_o, Y_o), with an initial velocity of v_o, having components $(v_o)x$ and $(v_o)y$, Fig. below . When air resistance is neglected, the only force acting on the projectile is its weight, which causes the projectile to have a constant downward acceleration of approximately

$$a_c = g = 9.81\ \text{m/s}^2 \text{ or } g = 32.2\ \text{ft/s}^2.^*$$



Horizontal Motion. Since $a_x = 0$, application of the constant acceleration equations, 12–4 to 12–6, yields

$$(\overset{+}{\rightarrow}) \qquad v = v_0 + a_c t; \qquad v_x = (v_0)_x$$
$$(\overset{+}{\rightarrow}) \qquad x = x_0 + v_0 t + \tfrac{1}{2} a_c t^2; \qquad x = x_0 + (v_0)_x t$$
$$(\overset{+}{\rightarrow}) \qquad v^2 = v_0^2 + 2a_c(x - x_0); \qquad v_x = (v_0)_x$$

Vertical Motion. Since the positive y axis is directed upward, then $a_y = -g$. Applying Eqs. 12–4 to 12–6, we get

$$(+\uparrow) \qquad v = v_0 + a_c t; \qquad v_y = (v_0)_y - gt$$
$$(+\uparrow) \qquad y = y_0 + v_0 t + \tfrac{1}{2} a_c t^2; \qquad y = y_0 + (v_0)_y t - \tfrac{1}{2} g t^2$$
$$(+\uparrow) \qquad v^2 = v_0^2 + 2a_c(y - y_0); \qquad v_y^2 = (v_0)_y^2 - 2g(y - y_0)$$

Recall that the last equation can be formulated on the basis of eliminating the time t from the first two equations, and therefore only two of the above three equations are independent of one another.

To summarize, problems involving the motion of a projectile can have at most three unknowns since only three independent equations can be written; that is, one equation in the horizontal direction and two in the vertical direction. Once V_x and V_y are obtained, the resultant velocity v, which is always tangent to the path, can be determined by the vector sum.

PROCEDURE OF ANALYSIS

Coordinate System .

- Establish the fixed x, y coordinate axes and sketch the trajectory of the particle. Between any two points on the path specify the given problem data and identify the three unknowns. In all cases the acceleration of gravity acts downward and equals 9.81 m/s^2 or 32.2 ft/s^2. The particle's initial and final velocities should be represented in terms of their x and y components.

• Remember that positive and negative position, velocity, and acceleration components always act in accordance with their associated coordinate directions. Kinematic Equations.

• Depending upon the known data and what is to be determined, a choice should be made as to which three of the following four equations should be applied between the two points on the path to obtain the most direct solution to the problem

Horizontal Motion.

- The *velocity* in the horizontal or x direction is *constant*, i.e., $v_x = (v_0)_x$, and

$$x = x_0 + (v_0)_x t$$

Vertical Motion.

- In the vertical or y direction *only two* of the following three equations can be used for solution.

$$v_y = (v_0)_y + a_c t$$

$$y = y_0 + (v_0)_y t + \frac{1}{2} a_c t^2$$

$$v_y^2 = (v_0)_y^2 + 2a_c(y - y_0)$$

For example, if the particle's final velocity v_y is not needed, then the first and third of these equations will not be useful.

EXA M P L E-1-

A sack slides off the ramp, shown in Fig. 12–21, with a horizontal velocity of 12 m/s. If the height of the ramp is 6 m from the floor, determine the time needed for the sack to strike the floor and the range R where sacks begin to pile up.

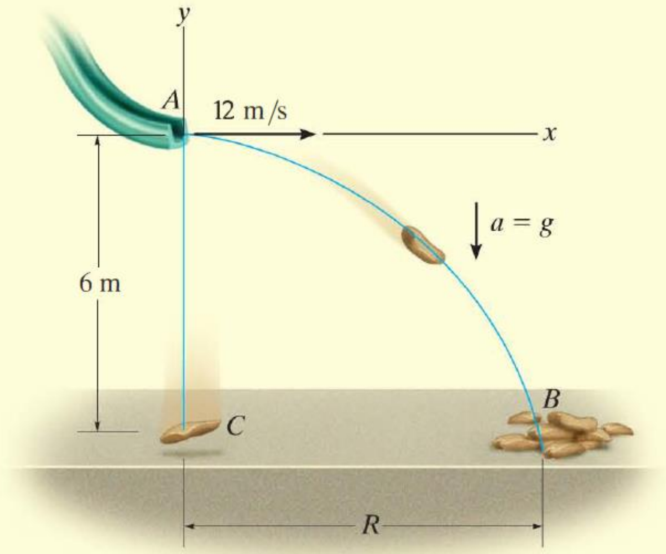


Fig. 12–21

SOLUTION

Coordinate System. The origin of coordinates is established at the beginning of the path, point A, Fig. 12–21. The initial velocity of a sack has components $(v_A)_x = 12$ m/s and $(v_A)_y = 0$. Also, between points A and B the acceleration is $a_y = -9.81 \text{ m/s}^2$. Since $(v_B)_x = (v_A)_x = 12$ m/s, the three unknowns are $(v_B)_y$, R, and the time of flight t_{AB}. Here we do not need to determine $(v_B)_y$.

Vertical Motion. The vertical distance from A to B is known, and therefore we can obtain a direct solution for t_{AB} by using the equation

$$(+\uparrow) \qquad y_B = y_A + (v_A)_y t_{AB} + \tfrac{1}{2} a_c t_{AB}^2$$

$$-6 \text{ m} = 0 + 0 + \tfrac{1}{2}(-9.81 \text{ m/s}^2) t_{AB}^2$$

$$t_{AB} = 1.11 \text{ s} \qquad \textit{Ans.}$$

Horizontal Motion. Since t_{AB} has been calculated, R is determined as follows:

$$(\xrightarrow{+}) \qquad x_B = x_A + (v_A)_x t_{AB}$$

$$R = 0 + 12 \text{ m/s } (1.11 \text{ s})$$

$$R = 13.3 \text{ m} \qquad \textit{Ans.}$$

EXA M P L E-2-

The chipping machine is designed to eject wood chips at $v_O = 25$ ft/s as shown in Fig. 12–22. If the tube is oriented at 30° from the horizontal, determine how high, h, the chips strike the pile if at this instant they land on the pile 20 ft from the tube.

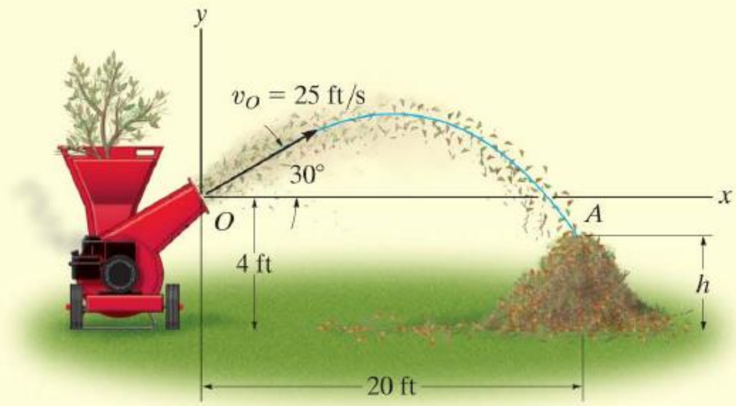


Fig. 12–22

SOLUTION

Coordinate System. When the motion is analyzed between points O and A, the three unknowns are the height h, time of flight t_{OA}, and vertical component of velocity $(v_A)_y$. [Note that $(v_A)_x = (v_O)_x$.] With the origin of coordinates at O, Fig. 12–22, the initial velocity of a chip has components of

$$(v_O)_x = (25 \cos 30°)\text{ ft/s} = 21.65\text{ ft/s}\rightarrow$$
$$(v_O)_y = (25 \sin 30°)\text{ ft/s} = 12.5\text{ ft/s}\uparrow$$

Also, $(v_A)_x = (v_O)_x = 21.65$ ft/s and $a_y = -32.2$ ft/s^2. Since we do not need to determine $(v_A)_y$, we have

Horizontal Motion.

$$(\overset{+}{\rightarrow}) \qquad x_A = x_O + (v_O)_x t_{OA}$$
$$20\text{ ft} = 0 + (21.65\text{ ft/s})t_{OA}$$
$$t_{OA} = 0.9238\text{ s}$$

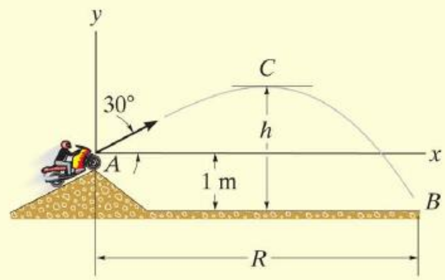
Vertical Motion. Relating t_{OA} to the initial and final elevations of a chip, we have

$$(+\uparrow) \qquad y_A = y_O + (v_O)_y t_{OA} + \tfrac{1}{2}a_c t_{OA}^2$$
$$(h - 4\text{ ft}) = 0 + (12.5\text{ ft/s})(0.9238\text{ s}) + \tfrac{1}{2}(-32.2\text{ ft/s}^2)(0.9238\text{ s})^2$$
$$h = 1.81\text{ ft}$$ *Ans.*

EXA M P L E-3-

The track for this racing event was designed so that riders jump off the slope at 30°, from a height of 1 m. During a race it was observed that the rider shown in Fig. 12–23*a* remained in mid air for 1.5 s. Determine the speed at which he was traveling off the ramp, the horizontal distance he travels before striking the ground, and the maximum height he attains. Neglect the size of the bike and rider.

(a)



(b)

Fig. 12–23

SOLUTION

Coordinate System. As shown in Fig. 12–23*b*, the origin of the coordinates is established at A. Between the end points of the path AB the three unknowns are the initial speed v_A, range R, and the vertical component of velocity $(v_B)_y$.

Vertical Motion. Since the time of flight and the vertical distance between the ends of the path are known, we can determine v_A.

$(+\uparrow)$
$$y_B = y_A + (v_A)_y t_{AB} + \tfrac{1}{2} a_c t_{AB}^2$$
$$-1\text{ m} = 0 + v_A \sin 30°(1.5\text{ s}) + \tfrac{1}{2}(-9.81\text{ m/s}^2)(1.5\text{ s})^2$$
$$v_A = 13.38\text{ m/s} = 13.4\text{ m/s} \qquad \textit{Ans.}$$

Horizontal Motion. The range R can now be determined.

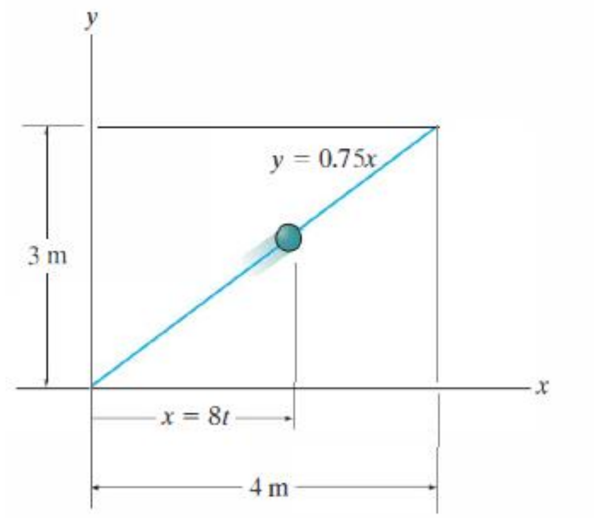
$(\overset{+}{\rightarrow})$
$$x_B = x_A + (v_A)_x t_{AB}$$
$$R = 0 + 13.38\cos 30°\text{ m/s}(1.5\text{ s})$$
$$= 17.4\text{ m} \qquad \textit{Ans.}$$

In order to find the maximum height h we will consider the path AC, Fig. 12–23*b*. Here the three unknowns are the time of flight t_{AC}, the horizontal distance from A to C, and the height h. At the maximum height $(v_C)_y = 0$, and since v_A is known, we can determine h *directly* without considering t_{AC} using the following equation.

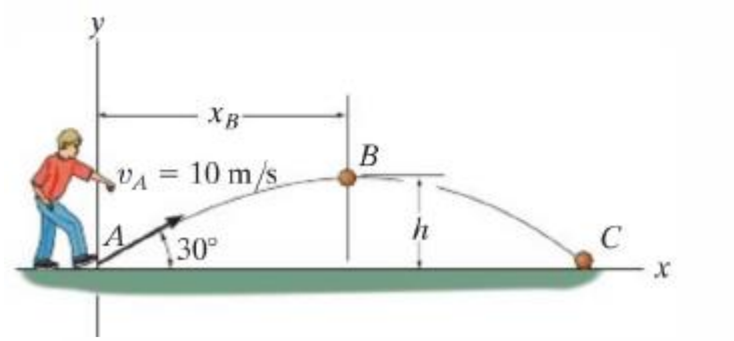
$$(v_C)_y^2 = (v_A)_y^2 + 2a_c[y_C - y_A]$$
$$0^2 = (13.38\sin 30°\text{ m/s})^2 + 2(-9.81\text{ m/s}^2)[(h - 1\text{ m}) - 0]$$
$$h = 3.28\text{ m} \qquad \textit{Ans.}$$

Problems:-

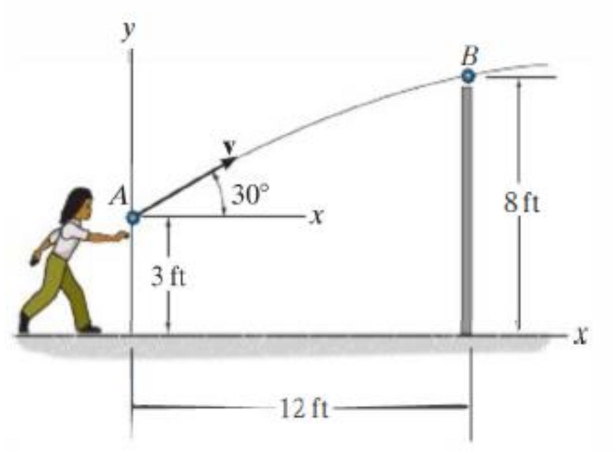
Q1/ A particle is traveling along the straight path. If its position along the x axis is x = (8t) m, where t is in seconds, determine its speed when t = 2 s.



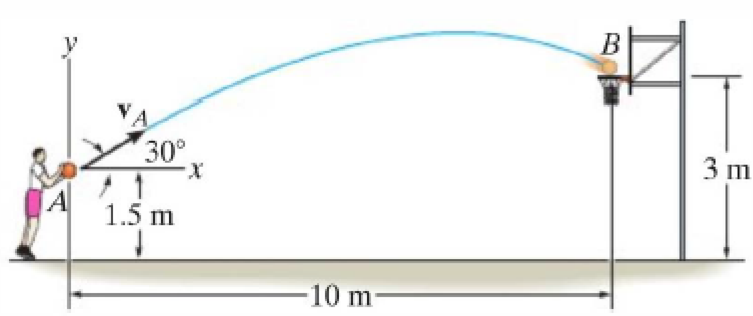
Q2/ The ball is kicked from point A with the initial velocity v_A = 10 m/s. Determine the range R, and the speed when the ball strikes the ground.



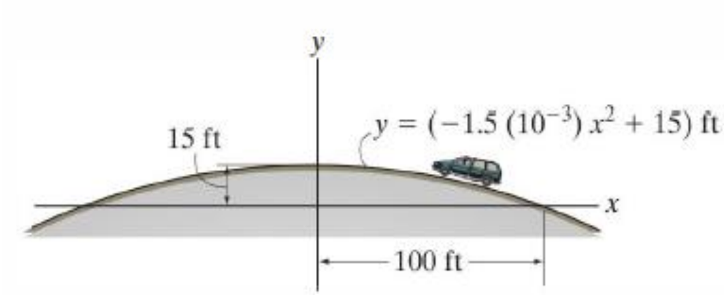
Q3/ A ball is thrown from A. If it is required to clear the wall at B, determine the minimum magnitude of its initial velocity V_A.



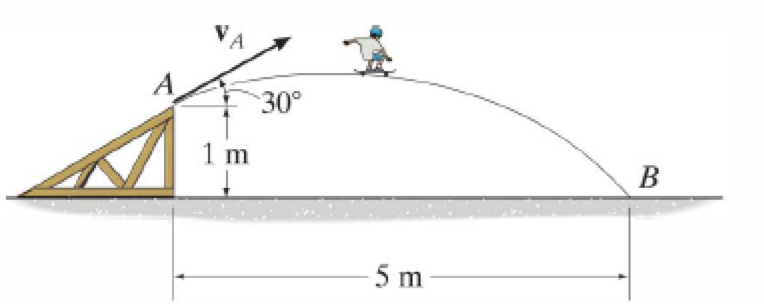
Q4/ Determine the speed at which the basketball at A must be thrown at the angle of 30° so that it makes it to the basket at B.



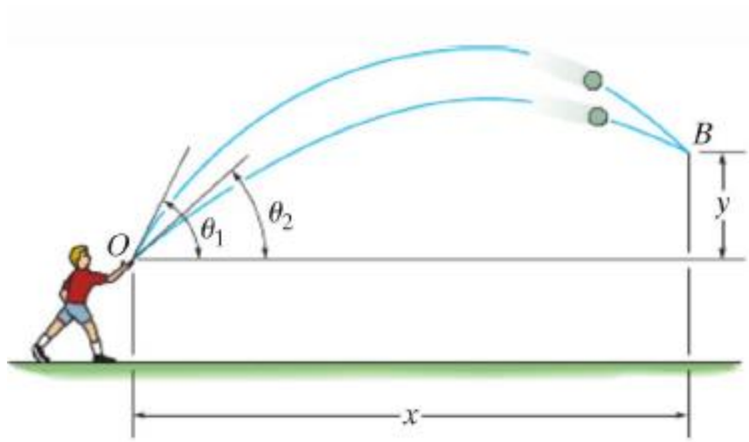
Q5/ The van travel over the hill described by $y = (-1.5(10^{-3})\,x^2 + 15)$ ft. If it has a constant speed of 75 ft/s, determine the x and y components of the van's velocity and acceleration when x = 5 0 ft.



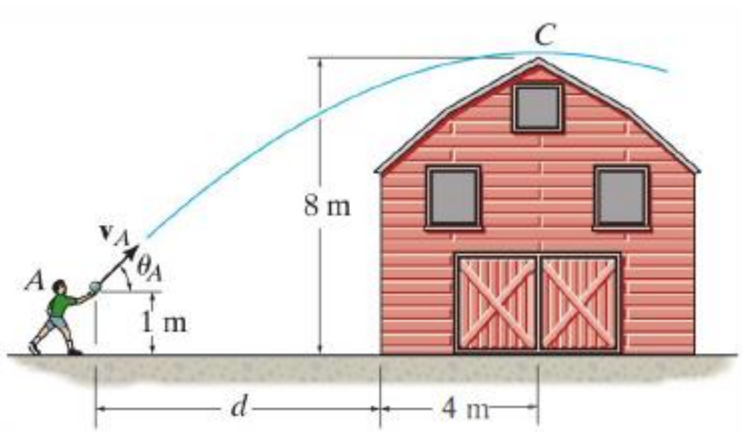
Q6/ The skateboard rider leaves the ramp at A with an initial velocity υ_A at a 30° angle. If he strikes the ground at B, determine v A and the time of flight



Q7/ A boy throws a ball at **O** in the air with a speed V_o at an angle θ_1. If he then throws another ball with the same speed Vo at an angle $\theta_2 < \theta_1$, determine the time between the throws so that the balls collide in mid air at B.

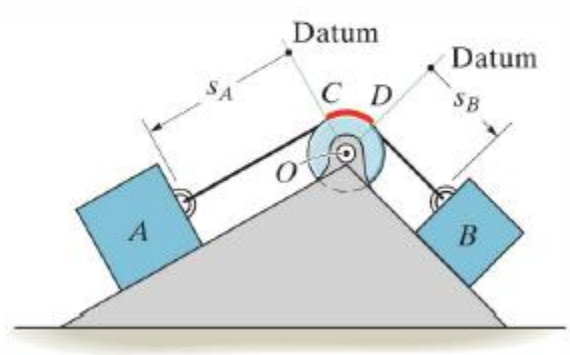


Q8/ The boy a t A attempts to throw a ball over the . roof of a barn with an initial speed of υ_A = 15 m/s. Determine the angle θ_A at which the ball must be thrown so that it reaches its maximum height at C. Also, find the distance d where the boy should stand to make the throw.



Absolute Dependent Motion Analysis of Two Particles

In some types of problems the motion of one particle will depend on the corresponding motion of another particle. This dependency commonly occurs if the particles, here represented by blocks, are interconnected by inextensible cords which are wrapped around pulleys. For example, the movement of block A downward along the inclined plane in Fig.



will cause a corresponding movement of block B up the other incline. We can show this mathematically by first specifying the location of the blocks using position coordinates SA and SB. Note that each of the coordinate axes is (1) measured from a fixed point (**O**) or fixed datum line, (2) measured along each inclined plane in the direction of motion of each block, and (3) has a positive sense from C to A and D to B. If the total cord length is l_T, the two position coordinates are related by the equation

$$s_A + l_{CD} + s_B = l_T$$

Here l_{CD} is the length of the cord passing over arc CD. Taking the time derivative of this expression, realizing that l_{CD} and l_T remain constant, while S_A and S_B measure the segments of the cord that change in length. We have

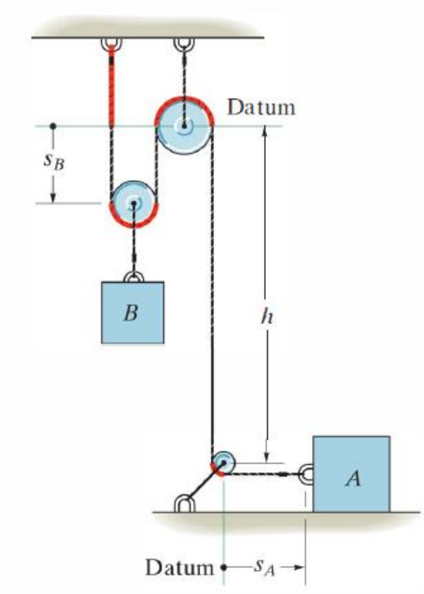
$$\frac{ds_A}{dt} + \frac{ds_B}{dt} = 0 \qquad \text{or} \qquad v_B = -v_A$$

The negative sign indicates that when block A has a velocity downward, i.e., in the direction of positive S_A, it causes a corresponding upward velocity of block B; i.e., B moves in the negative S_B direction. In a similar manner, time differentiation of the velocities yields the relation between the accelerations, i.e.,

$$a_B = -a_A$$

A more complicated example is shown in Fig. a. In this case, the position of block A is specified by S_A, and the position of the end of the cord from which block B is suspended is defined by S_B.

As above, we have chosen position coordinates which (1) have their origin at fixed points or datums, (2) are measured in the direction of motion of each block, and (3) are positive to the right for S_A and positive downward for S_B. During the motion, the length of the red colored segments of the cord in Fig a. remains constant. If *l* represents the total length of cord minus these segments, then the position coordinates can be related by the equation



(a)

$$2s_B + h + s_A = l$$

Since *l* and h are constant during the motion, the two time derivatives yield

$$2v_B = -v_A \qquad 2a_B = -a_A$$

Hence, when B moves downward ($+S_B$), A moves to the left ($-S_A$) with twice the motion

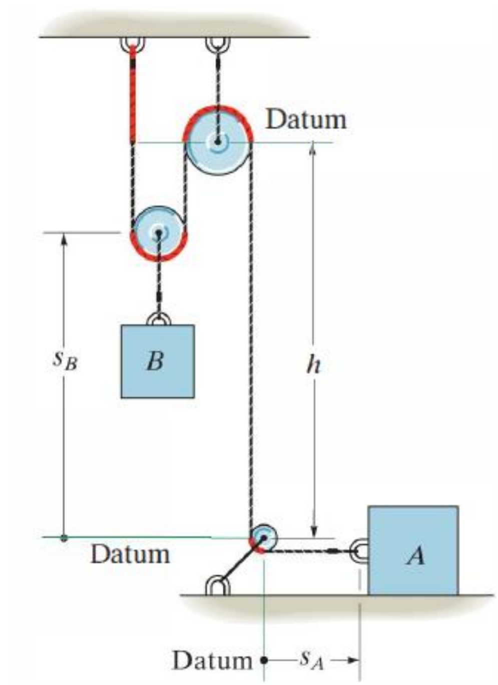
This example can also be worked by defining the position of block B from the center of the bottom pulley (a fixed point), Fig. b. In this case

$$2(h - s_B) + h + s_A = l$$

Time differentiation yields

$$2v_B = v_A \qquad 2a_B = a_A$$

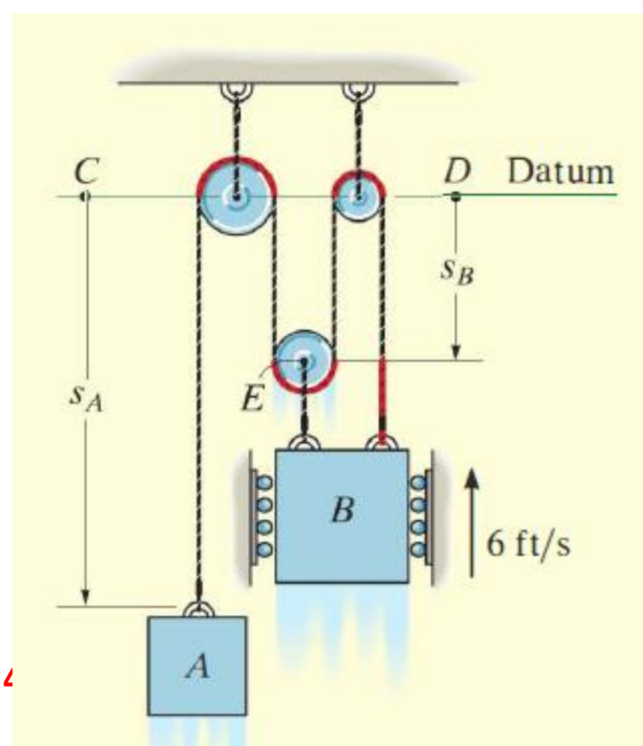
Here the signs are the same. Why?



(b)

EXA M P L E-1-

Determine the speed of block A in Fig. if block B has an upward speed of 6 ft/s.



SOLUTION

Position-Coordinate Equation. There is *one cord* in this system having segments which change length. Position coordinates s_A and s_B will be used since each is measured from a fixed point (C or D) and extends along each block's *path of motion*. In particular, s_B is directed to point E since motion of B and E is the *same*.

The red colored segments of the cord in Fig. 12–38 remain at a constant length and do not have to be considered as the blocks move. The remaining length of cord, l, is also constant and is related to the changing position coordinates s_A and s_B by the equation

$$s_A + 3s_B = l$$

Time Derivative. Taking the time derivative yields

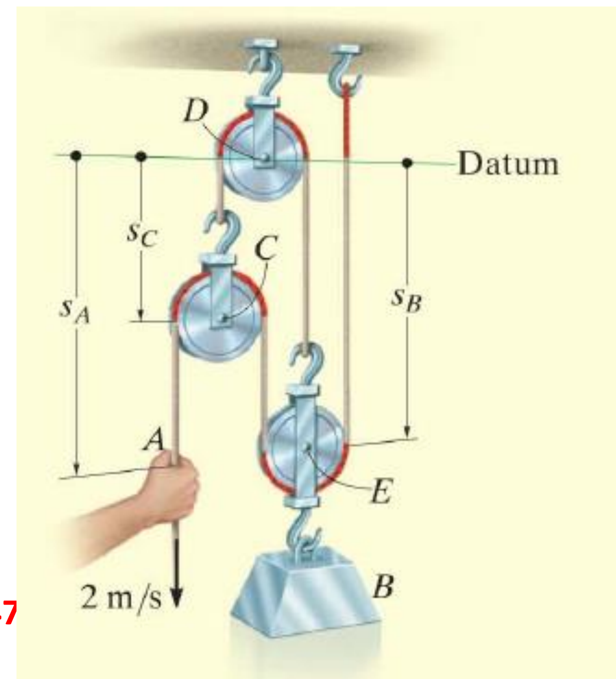
$$v_A + 3v_B = 0$$

so that when $v_B = -6$ ft/s (upward),

$$v_A = 18 \text{ ft/s}\downarrow \qquad \textit{Ans.}$$

EXA M P L E-2-

Determine the speed of block B in Fig. if the end of the cord at A is pulled down with a speed of 2 m/s.



SOLUTION

Position-Coordinate Equation. The position of point A is defined by s_A, and the position of block B is specified by s_B since point E on the pulley will have the *same motion* as the block. Both coordinates are measured from a horizontal datum passing through the *fixed* pin at pulley D. Since the system consists of *two* cords, the coordinates s_A and s_B cannot be related directly. Instead, by establishing a third position coordinate, s_C, we can now express the length of one of the cords in terms of s_B and s_C, and the length of the other cord in terms of s_A, s_B, and s_C.

Excluding the red colored segments of the cords in Fig. 12–40, the remaining constant cord lengths l_1 and l_2 (along with the hook and link dimensions) can be expressed as

$$s_C + s_B = l_1$$

$$(s_A - s_C) + (s_B - s_C) + s_B = l_2$$

Time Derivative. The time derivative of each equation gives

$$v_C + v_B = 0$$

$$v_A - 2v_C + 2v_B = 0$$

Eliminating v_C, we obtain

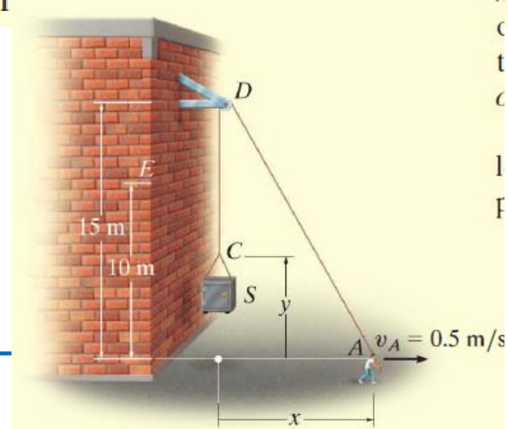
$$v_A + 4v_B = 0$$

so that when $v_A = 2$ m/s (downward),

$$v_B = -0.5 \text{ m/s} = 0.5 \text{ r}$$

EXA M P L E-2-

A man at A is hoisting a safe S as shown in Fig. by walking to the right with a constant velocity VA=0.5m/s. Determine the velocity and acceleration of the safe when it reaches



the elevation of 10 m. The rope is 30 m long and passes over a small pulley at D.

SOLUTION

Position-Coordinate Equation. This problem is unlike the previous examples since rope segment *DA* changes *both direction and magnitude*. However, the ends of the rope, which define the positions of *S* and *A*, are specified by means of the *x* and *y* coordinates since they must be measured from a fixed point and *directed along the paths of motion* of the ends of the rope.

The *x* and *y* coordinates may be related since the rope has a fixed length $l = 30$ m, which at all times is equal to the length of segment *DA* plus *CD*. Using the Pythagorean theorem to determine l_{DA}, we have $l_{DA} = \sqrt{(15)^2 + x^2}$; also, $l_{CD} = 15 - y$. Hence,

$$l = l_{DA} + l_{CD}$$

's

$$30 = \sqrt{(15)^2 + x^2} + (15 - y)$$

$$y = \sqrt{225 + x^2} - 15 \qquad (1)$$

Time Derivatives. Taking the time derivative, using the chain rule (see Appendix C), where $v_S = dy/dt$ and $v_A = dx/dt$, yields

$$v_S = \frac{dy}{dt} = \left[\frac{1}{2}\frac{2x}{\sqrt{225 + x^2}}\right]\frac{dx}{dt}$$

$$= \frac{x}{\sqrt{225 + x^2}}v_A \qquad (2)$$

At $y = 10$ m, x is determined from Eq. 1, i.e., $x = 20$ m. Hence, from Eq. 2 with $v_A = 0.5$ m/s,

$$v_S = \frac{20}{\sqrt{225 + (20)^2}}(0.5) = 0.4\text{m/s} = 400\text{mm/s} \uparrow \qquad \textit{Ans.}$$

The acceleration is determined by taking the time derivative of Eq. 2. Since v_A is constant, then $a_A = dv_A/dt = 0$, and we have

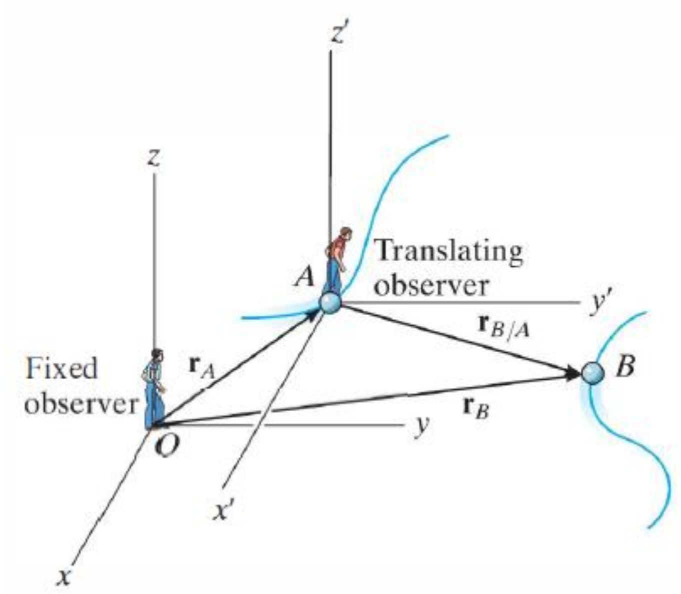
$$a_S = \frac{d^2y}{dt^2} = \left[\frac{-x(dx/dt)}{(225 + x^2)^{3/2}}\right]xv_A + \left[\frac{1}{\sqrt{225 + x^2}}\right]\left(\frac{dx}{dt}\right)v_A + \left[\frac{1}{\sqrt{225 + x^2}}\right]x\frac{dv_A}{dt} = \frac{225v_A^2}{(225 + x^2)^{3/2}}$$

At $x = 20$ m, with $v_A = 0.5$ m/s, the acceleration becomes

$$a_S = \frac{225(0.5\text{ m/s})^2}{[225 + (20\text{ m})^2]^{3/2}} = 0.00360\text{ m/s}^2 = 3.60\text{ mm/s}^2 \uparrow \quad \textit{Ans.}$$

Relative-Motion of Two Particles Using Translating Axes

Position. Consider particles A and B, which move along the arbitrary paths shown in Fig. 12-42. The absolute position of each particle, r_A and r_B , is measured from the common origin **O** of the fixed x,y,z reference frame. The origin of a second frame of reference x', y ' , z ' is attached to and move with particle A. The axes of this frame are only relative to the fixed frame. The position of B measured relative to A is denoted by the relative-position vector $r_{B/A'}$ Using vector addition, the three vectors shown in Fig. can be related by the equation



$$\mathbf{r}_B = \mathbf{r}_A + \mathbf{r}_{B/A}$$

Velocity. An equation that relates the velocities of the particles is determined by taking the time derivative of the above equation; i.e.,

$$\mathbf{v}_B = \mathbf{v}_A + \mathbf{v}_{B/A}$$

Here $V_B = dr_B/ dt$ and $vA = dr_A/ dt$ refer to absolute velocities, since they are observed from the fixed frame; whereas the relative velocity $v_{B/A} = dr_{B/A}/dt$ is observed from the translating frame. It is important to note that since the x', y', z ' axes translate, the components of $r_{B/A}$ will not change direction and therefore the time derivative of these components will only have to account for the change in their magnitudes. Above Equation therefore states that the velocity of B is equal to the velocity of A plus (vectorially) the velocity of "B with respect to A ," as measured by the translating observer fixed in the x', y', z ' reference frame.

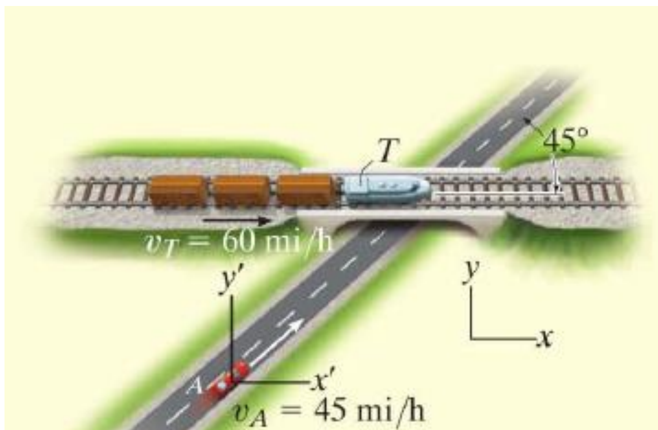
Acceleration. The time derivative of the Eq. $\mathbf{v}_B = \mathbf{v}_A + \mathbf{v}_{B/A}$ yields a similar vector relation between the absolute and relative accelerations of particles A and B

$$\mathbf{a}_B = \mathbf{a}_A + \mathbf{a}_{B/A}$$

Here $\mathbf{a}_{B/A}$ is the acceleration of B as seen by the observer located at A and translating with the x', y', z' reference frame.*

EXAMPLE-1-

A train travels at a constant speed of 60 mi/h, crosses over a road as shown in Fig. a. If the automobile A is traveling at 45 mi/h along the road, determine the magnitude and direction of the velocity of the train relative to the automobile



(a)

SOLUTION I

Vector Analysis. The relative velocity $\mathbf{v}_{T/A}$ is measured from the translating x', y' axes attached to the automobile, Fig. 12–43*a*. It is determined from $\mathbf{v}_T = \mathbf{v}_A + \mathbf{v}_{T/A}$. Since $\mathbf{v}_T$ and $\mathbf{v}_A$ are known in *both* magnitude and direction, the unknowns become the x and y components of $\mathbf{v}_{T/A}$. Using the x, y axes in Fig. 12–43*a*, we have

$$\mathbf{v}_T = \mathbf{v}_A + \mathbf{v}_{T/A}$$

$$60\mathbf{i} = (45 \cos 45°\mathbf{i} + 45 \sin 45°\mathbf{j}) + \mathbf{v}_{T/A}$$

$$\mathbf{v}_{T/A} = \{28.2\mathbf{i} - 31.8\mathbf{j}\} \text{ mi/h} \qquad \textit{Ans.}$$

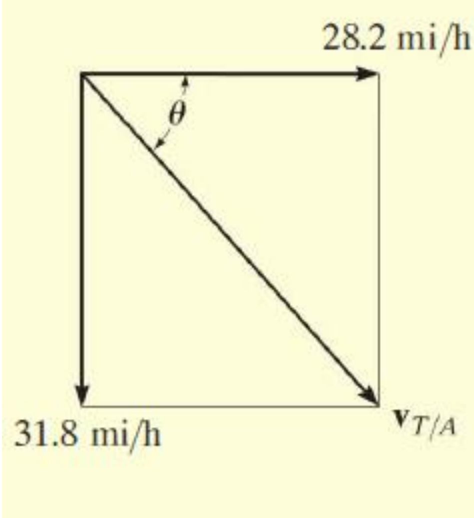
The magnitude of $\mathbf{v}_{T/A}$ is thus

$$v_{T/A} = \sqrt{(28.2)^2 + (-31.8)^2} = 42.5 \text{ mi/h} \qquad \textit{Ans.}$$

From the direction of each component, Fig. 12–43*b*, the direction of $\mathbf{v}_{T/A}$ is

$$\tan\theta = \frac{(v_{T/A})_y}{(v_{T/A})_x} = \frac{31.8}{28.2}$$

$$\theta = 48.5° \qquad \textit{Ans.}$$



(b)

Note that the vector addition shown in Fig. b indicates the correct sense for $V_{T/A}$' This figure anticipates the answer and can be used to check it.

SOLUTION II

Scalar Analysis. The unknown components of $\mathbf{v}_{T/A}$ can also be determined by applying a scalar analysis. We will assume these components act in the *positive* *x* and *y* directions. Thus,

$$\mathbf{v}_T = \mathbf{v}_A + \mathbf{v}_{T/A}$$

$$\begin{bmatrix} 60 \text{ mi/h} \\ \rightarrow \end{bmatrix} = \begin{bmatrix} 45 \text{ mi/h} \\ \measuredangle^{45^\circ} \end{bmatrix} + \begin{bmatrix} (v_{T/A})_x \\ \rightarrow \end{bmatrix} + \begin{bmatrix} (v_{T/A})_y \\ \uparrow \end{bmatrix}$$

Resolving each vector into its *x* and *y* components yields

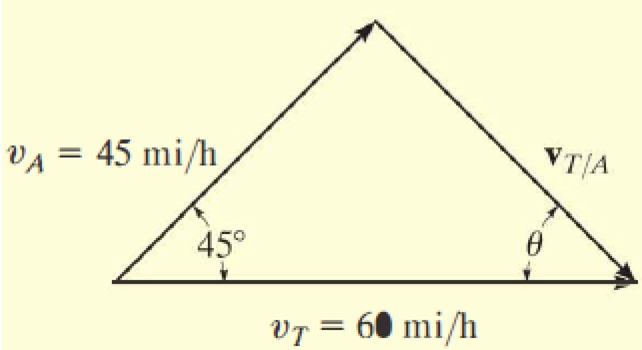
$$(\overset{+}{\rightarrow}) \qquad 60 = 45 \cos 45^\circ + (v_{T/A})_x + 0$$

$$(+\uparrow) \qquad 0 = 45 \sin 45^\circ + 0 + (v_{T/A})_y$$

Solving, we obtain the previous results,

$$(v_{T/A})_x = 28.2 \text{ mi/h} = 28.2 \text{ mi/h} \rightarrow$$

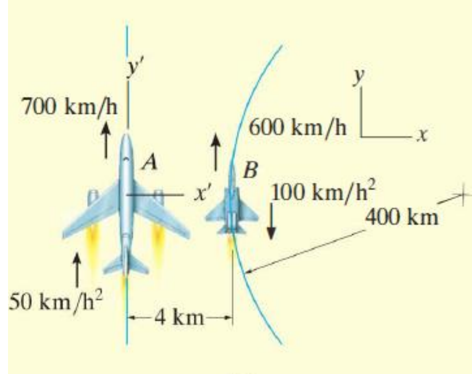
$$(v_{T/A})_y = -31.8 \text{ mi/h} = 31.8 \text{ mi/h} \downarrow$$



(c)

EXAMPLE-2-

Plane A in Fig. a is flying along a straight-line path, whereas plane B is flying along a circular path having a radius of curvature of PB = 400 km. Determine the velocity and acceleration of B as measured by the pilot of A.



(a)

SO LUTI O N

Velocity. The origin of the x and y axes are located at an arbitrary fixed point. Since the motion relative to plane A is to be determined, the translating frame of reference x ', y' is attached to it, Fig. a .Applying the relative-velocity equation in scalar form since the velocity vectors of both planes are parallel at the instant shown, we have

$$(+\uparrow) \qquad v_B = v_A + v_{B/A}$$

$$600\text{ km/h} = 700\text{ km/h} + v_{B/A}$$

$$v_{B/A} = -100\text{ km/h} = 100\text{ km/h}\downarrow \qquad \textit{Ans.}$$

The vector addition is shown in Fig. 12–44*b*.

Acceleration. Plane *B* has both tangential and normal components of acceleration since it is flying along a *curved path*. From Eq. 12–20, the magnitude of the normal component is

$$(a_B)_n = \frac{v_B^2}{\rho} = \frac{(600\text{ km/h})^2}{400\text{ km}} = 900\text{ km/h}^2$$

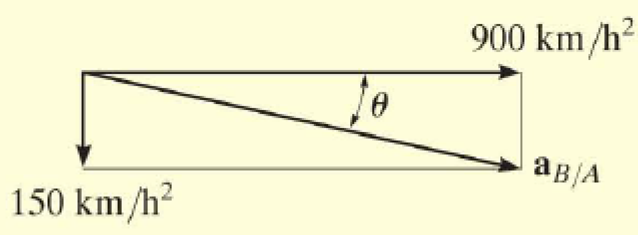
Applying the relative-acceleration equation gives

$$\mathbf{a}_B = \mathbf{a}_A + \mathbf{a}_{B/A}$$

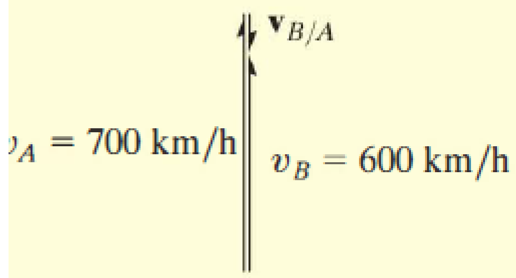
$$900\mathbf{i} - 100\mathbf{j} = 50\mathbf{j} + \mathbf{a}_{B/A}$$

Thus,

$$\mathbf{a}_{B/A} = \{900\mathbf{i} - 150\mathbf{j}\}\text{ km/h}^2$$



(c)



(b)

Acceleration. Plane B has both tangential and normal components of acceleration since it is flying along a *curved path*. From Eq. 12–20, the magnitude of the normal component is

$$(a_B)_n = \frac{v_B^2}{\rho} = \frac{(600\ \text{km/h})^2}{400\ \text{km}} = 900\ \text{km/h}^2$$

Applying the relative-acceleration equation gives

$$\mathbf{a}_B = \mathbf{a}_A + \mathbf{a}_{B/A}$$

$$900\mathbf{i} - 100\mathbf{j} = 50\mathbf{j} + \mathbf{a}_{B/A}$$

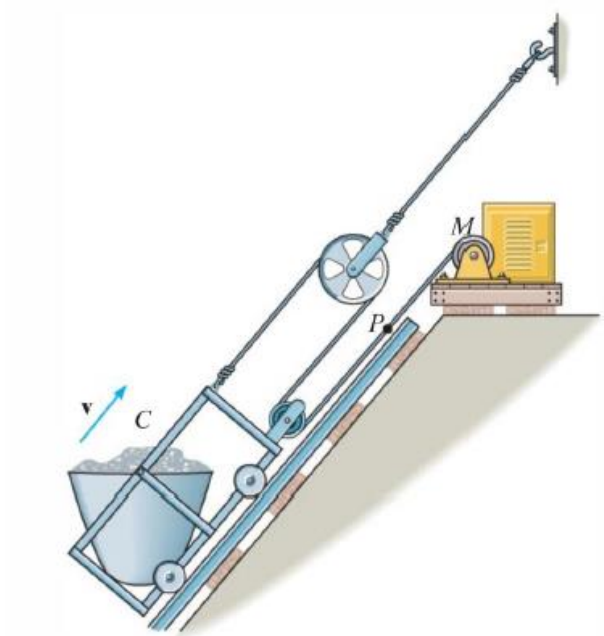
Thus,

$$\mathbf{a}_{B/A} = \{900\mathbf{i} - 150\mathbf{j}\}\ \text{km/h}^2$$

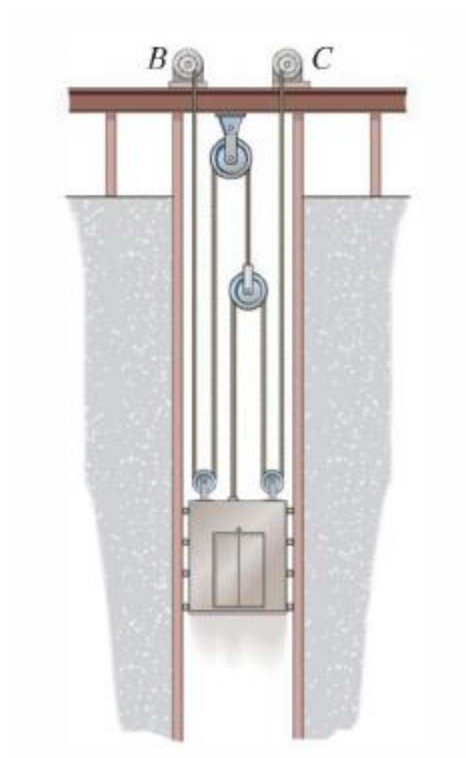
From Fig. 12–44*c*, the magnitude and direction of $\mathbf{a}_{B/A}$ are therefore

$$a_{B/A} = 912\ \text{km/h}^2 \qquad \theta = \tan^{-1}\frac{150}{900} = 9.46^\circ \quad \measuredangle \qquad \textit{Ans.}$$

PROBLRMS

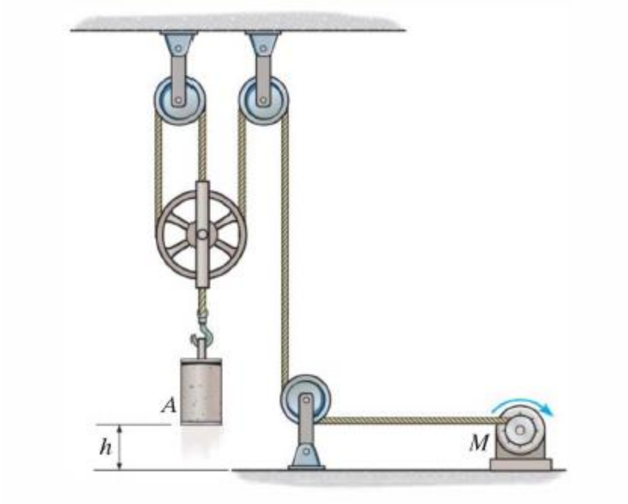


Q1/ The mine car C is being pulled up the incline using the motor M and the rope-and-pulley arrangement shown. Determine the speed v_p at which a point P on the cable must be traveling toward the motor to move the car up the plane with a constant speed of $\upsilon = 2$ m/s.

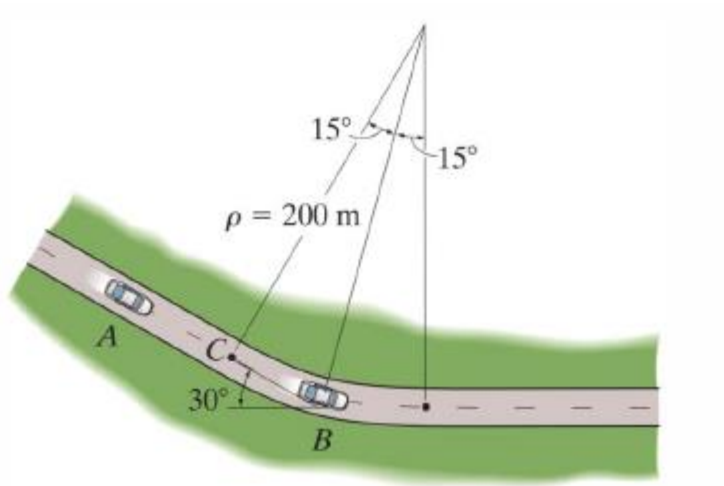


Q2/ Determine the speed of the elevator if each motor draws in the cable with a constant speed of 5 m/s.

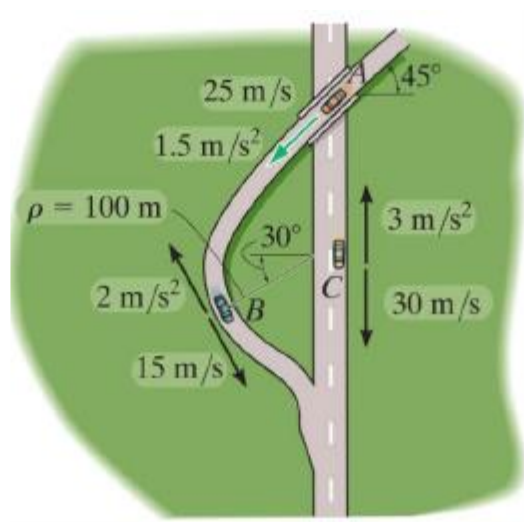
Q3/ If the rope is drawn towards the motor M at a speed of $V_M = (5t^{3/2})$ m/s, where t is in seconds, determine the speed of cylinder A when t = 1 s.



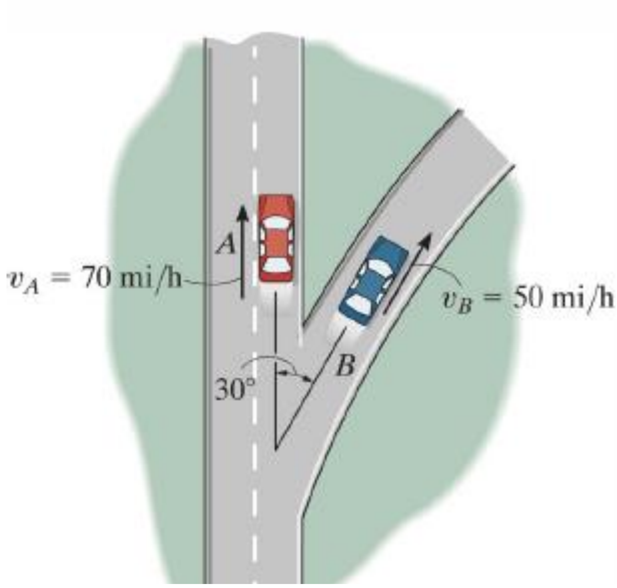
Q4/ At the instant shown, car A travels along the straight portion of the road with a speed of 25 m/s. At this same instant car B travels along the circular portion of the road with a speed of 15 m/s. Determine the velocity of car B relative to car A.



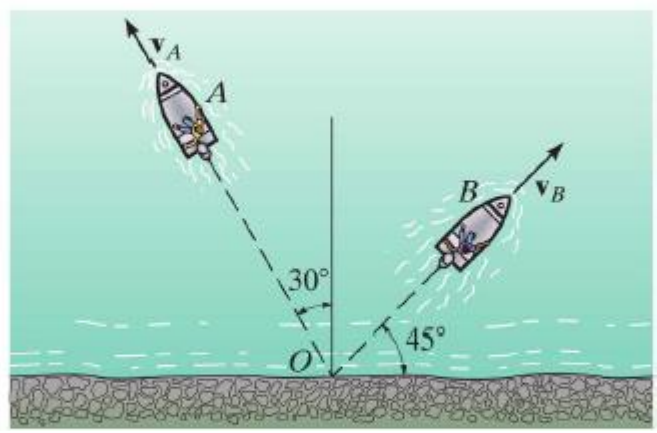
Q5/ Car B is traveling along the curved road with a speed of 15 m/s while decreasing its speed at 2 m/s². At this same instant car C is traveling along the straight road with a speed of 30 m/s while decelerating at 3 m/s². Determine the velocity and acceleration of car B relative to car C.



Q6/ At the instant shown, cars A and B travel at speeds of 70 mi/h and 50 mi/h, respectively. If B is decreasing its speed at 1400 mi/h^2 while A is increasing its speed at 800 mi/h^2, determine the acceleration of B with respect to A. Car B moves along a curve having a radius of curvature of 0.7 mi?



Q7/ Two boats leave the shore at the same time and travel in the directions shown.
If v_A = 20 ft/s and V_B = 15 ft/s, determine the velocity of boat A with respect to boat B. How long after leaving the shore will the boats be 800 ft apart?

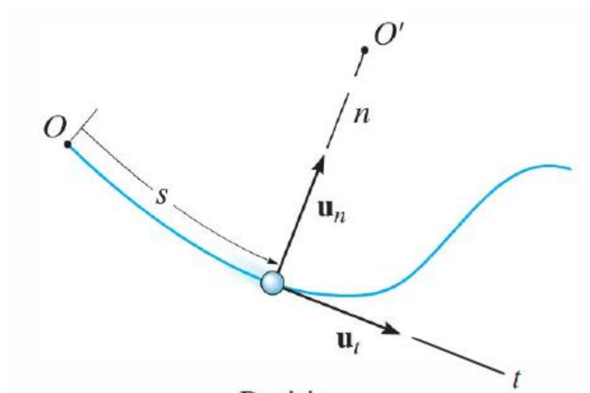


Curvilinear Motion Normal and Tangential Components

When the path along which a particle travels is known, then it is often convenient to describe the motion using n and t coordinate axes which act normal and tangent to the path, respectively, and at the instant considered have their origin located at the particle

Planar Motion . Consider the particle shown in Fig. a, which moves in a plane along a fixed curve, such that at a given instant it is at position s, measured from point O.

We will now consider a coordinate system that has its origin at a fixed point on the curve, and at the instant considered this origin happens to coincide with the location of the particle. The ***t*** axis is tangent to the curve at the point and is positive in the direction of increasing s. We will designate this positive direction with the unit vector U_t

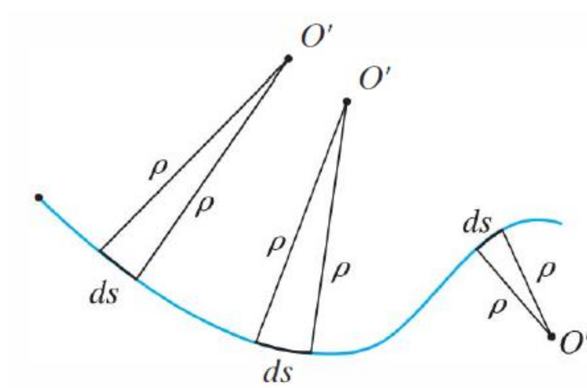


Position

(a)

• A unique choice for the normal axis can be made by noting that geometrically the curve is constructed from a series of differential arc segments ds, Fig. b.

Each segment ds is formed from the arc of an associated circle having a radius of curvature **ρ** (rho) and center of curvature 0 ' . The normal

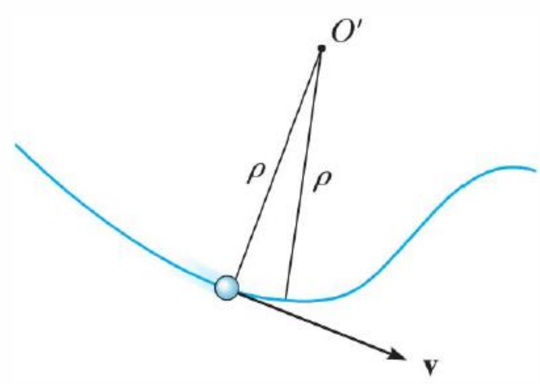


Radius of curvature

(b)

axis n is perpendicular to the t axis with its positive sense directed toward the center of curvature 0 ' , Fig. a. This positive direction, which is always on the concave side of the curve, will be designated by the unit vector U_n ' The plane which contains the *n* and *t* axes is referred to as the embracing or osculating plane, and in this case it is fixed in the plane of motion.

Velocity. Since the particle moves, *s* is a function of time. the particle's velocity v has a direction that is always tangent to the path, Fig. c, and a magnitude that is determined by taking the time derivative of the path function s = s(t), i.e., v = ds/dt (Hence)



Velocity
(c)

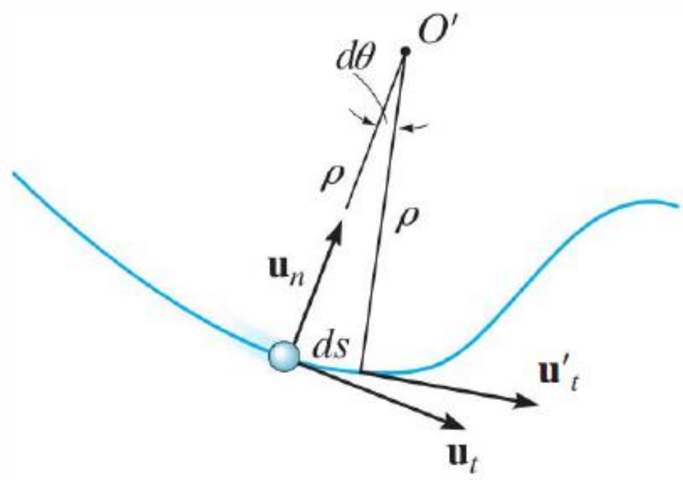
$$\mathbf{v} = v\mathbf{u}_t$$

$$v = \dot{s}$$

Acceleration. The acceleration of the particle is the time rate of change of the velocity. Thus,

$$\mathbf{a} = \dot{\mathbf{v}} = \dot{v}\mathbf{u}_t + v\dot{\mathbf{u}}_t$$

In order to determine the time derivative U_t , note that as the particle moves along the arc ds in time dt, U_t preserves its magnitude of unity;



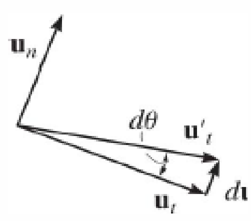
(d)

however, its direction changes, and becomes u; , Fig.d. As shown in Fig. e, we require $U_t = U_f + du_t$. Here du_t stretches between the arrowheads of U_t and u_t, which lie on an infinitesimal arc of radius $U_f = 1$. Hence, dU_t has a magnitude of $dU_f = (1)\,d\theta$, and its direction is defined by U_n' Consequently, $dU_t = d\theta\ U_n$, and therefore the time derivative becomes $U_t = \theta U_n$. Since $ds = \rho d\ d\theta$, Fig. d, then $d\theta = s/\rho$, and therefore.

$$\dot{\mathbf{u}}_t = \dot{\theta}\mathbf{u}_n = \frac{\dot{s}}{\rho}\mathbf{u}_n = \frac{v}{\rho}\mathbf{u}_n$$

$$\mathbf{a} = a_t\mathbf{u}_t + a_n\mathbf{u}_n$$

can be written as the sum of its two components,



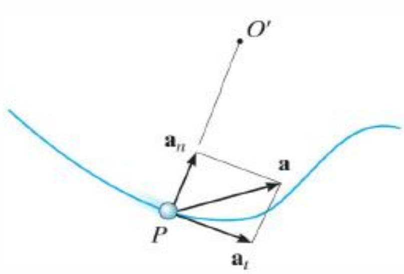
(e)

$$\mathbf{a} = a_t\mathbf{u}_t + a_n\mathbf{u}_n$$

where

$$a_t = \dot{v} \quad \text{or} \quad a_t ds = v\,dv$$

and



Acceleration

(f)

$$a_n = \frac{v^2}{\rho}$$

These two mutually perpendicular components are shown in Fig. f. Therefore, the magnitude of acceleration is the positive value of

$$a = \sqrt{a_t^2 + a_n^2}$$

PROCEDURE OF ANALYSIS

Velocity.

- The particle's *velocity* is always tangent to the path.
- The magnitude of velocity is found from the time derivative of the path function.

$$v = \dot{s}$$

Tangential Acceleration.

- The tangential component of acceleration is the result of the time rate of change in the *magnitude* of velocity. This component acts in the positive s direction if the particle's speed is increasing or in the opposite direction if the speed is decreasing.
- The relations between a_t, v, t and s are the same as for rectilinear motion, namely,

$$a_t = \dot{v} \quad a_t\,ds = v\,dv$$

- If a_t is constant, $a_t = (a_t)_c$, the above equations, when integrated, yield

$$s = s_0 + v_0 t + \tfrac{1}{2}(a_t)_c t^2$$
$$v = v_0 + (a_t)_c t$$
$$v^2 = v_0^2 + 2(a_t)_c(s - s_0)$$

Normal Acceleration.

- The normal component of acceleration is the result of the time rate of change in the *direction* of the velocity. This component is *always* directed toward the center of curvature of the path, i.e., along the positive n axis.
- The magnitude of this component is determined from

$$a_n = \frac{v^2}{\rho}$$

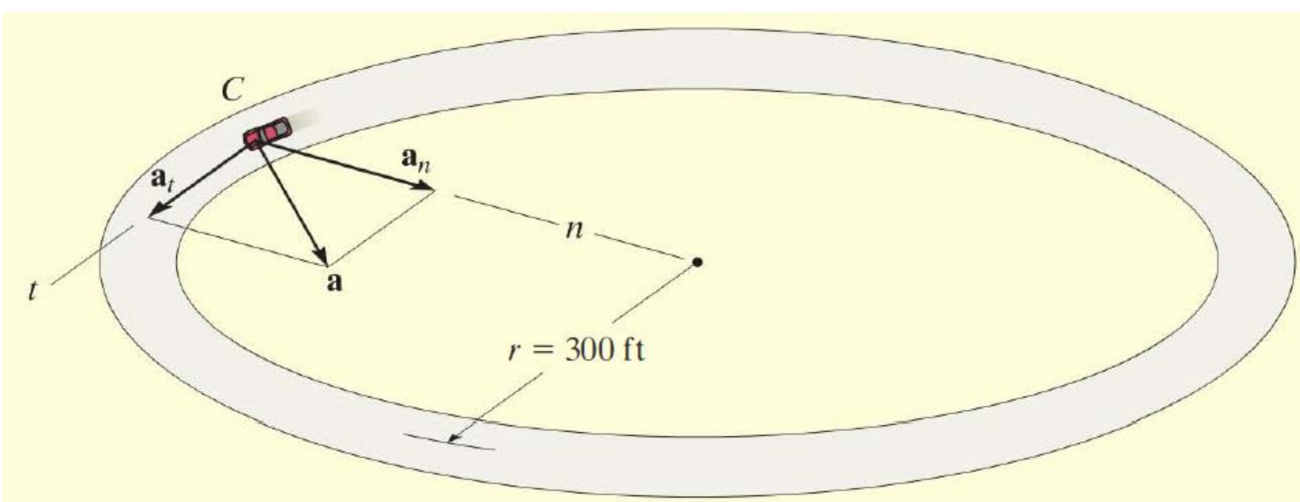
- If the path is expressed as $y = f(x)$, the radius of curvature ρ at any point on the path is determined from the equation

$$\rho = \frac{[1 + (dy/dx)^2]^{3/2}}{|d^2y/dx^2|}$$

The derivation of this result is given in any standard calculus text.

EXA M P L E -1-

A race car C travels around the horizontal circular track that has a radius of 300 ft, Fig. If the car increases its speed at a constant rate of 7 ft/S^2, starting from rest, determine the time needed for it to reach an acceleration of 8 ft/S^2. What is its speed at this instant



SOLUTION

Coordinate System. The origin of the n and t axes is coincident with the car at the instant considered. The t axis is in the direction of motion, and the positive n axis is directed toward the center of the circle. This coordinate system is selected since the path is known.

Acceleration. The magnitude of acceleration can be related to its components using $a = \sqrt{a_t^2 + a_n^2}$. Here $a_t = 7\ \text{ft/s}^2$. Since $a_n = v^2/\rho$, the velocity as a function of time must be determined first.

$$v = v_0 + (a_t)_c t$$
$$v = 0 + 7t$$

Thus

$$a_n = \frac{v^2}{\rho} = \frac{(7t)^2}{300} = 0.163t^2\ \text{ft/s}^2$$

The time needed for the acceleration to reach $8\ \text{ft/s}^2$ is therefore

$$a = \sqrt{a_t^2 + a_n^2}$$
$$8\ \text{ft/s}^2 = \sqrt{(7\ \text{ft/s}^2)^2 + (0.163t^2)^2}$$

Solving for the positive value of t yields

$$0.163t^2 = \sqrt{(8\ \text{ft/s}^2)^2 - (7\ \text{ft/s}^2)^2}$$
$$t = 4.87\ \text{s} \qquad \textit{Ans.}$$

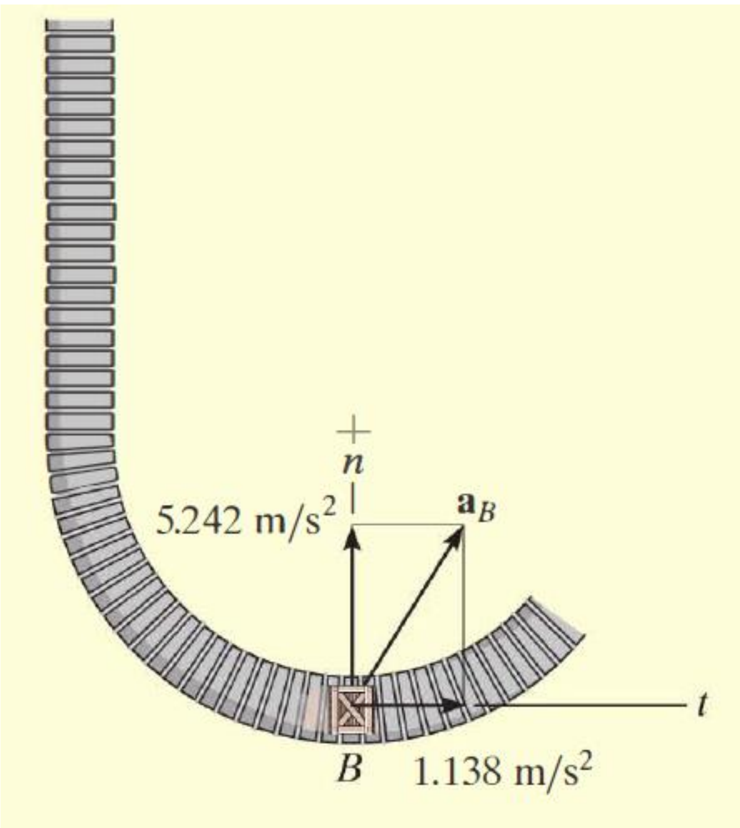
Velocity. The speed at time $t = 4.87$ s is

$$v = 7t = 7(4.87) = 34.1\ \text{ft/s} \qquad \textit{Ans.}$$

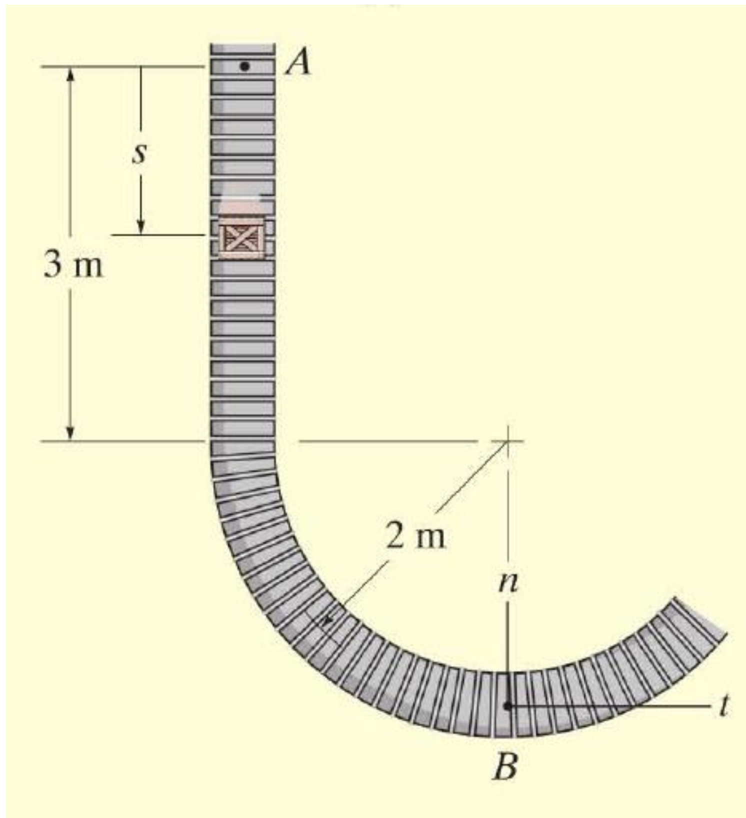
EXA M P L E -1-

EXA M P L E -2-

The boxes in Fig. 12-29a travel along the industrial conveyor. If a box as in Fig. b starts from rest at A and increases its speed such that at = (0.2t) m/s^2
, where t is in seconds, determine the magnitude of its acceleration when it arrives at point B?



(c)



(b)

SOLUTION

Coordinate System. The position of the box at any instant is defined from the fixed point A using the position or path coordinate s, Fig. 12–29b. The acceleration is to be determined at B, so the origin of the n, t axes is at this point.

Acceleration. To determine the acceleration components $a_t = \dot{v}$ and $a_n = v^2/\rho$, it is first necessary to formulate v and $\dot{v}$ so that they may be evaluated at B. Since $v_A = 0$ when $t = 0$, then

$$a_t = \dot{v} = 0.2t \qquad (1)$$

$$\int_0^v dv = \int_0^t 0.2t\, dt$$

$$v = 0.1t^2 \qquad (2)$$

The time needed for the box to reach point B can be determined by realizing that the position of B is $s_B = 3 + 2\pi(2)/4 = 6.142$ m, Fig. 12–29b, and since $s_A = 0$ when $t = 0$ we have

$$v = \frac{ds}{dt} = 0.1t^2$$

$$\int_0^{6.142\text{ m}} ds = \int_0^{t_B} 0.1t^2 dt$$

$$6.142\text{ m} = 0.0333t_B^3$$

$$t_B = 5.690\text{s}$$

Substituting into Eqs. 1 and 2 yields

$$(a_B)_t = \dot{v}_B = 0.2(5.690) = 1.138\text{ m/s}^2$$

$$v_B = 0.1(5.69)^2 = 3.238\text{ m/s}$$

At B, $\rho_B = 2$ m, so that

$$(a_B)_n = \frac{v_B^2}{\rho_B} = \frac{(3.238\text{ m/s})^2}{2\text{ m}} = 5.242\text{ m/s}^2$$

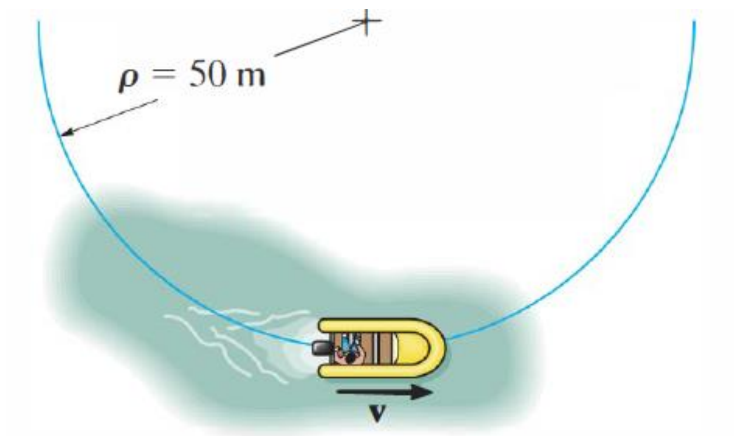
The magnitude of $\mathbf{a}_B$, Fig. 12–29c, is therefore

$$a_B = \sqrt{(1.138\text{ m/s}^2)^2 + (5.242\text{ m/s}^2)^2} = 5.36\text{ m/s}^2 \qquad \textit{Ans.}$$

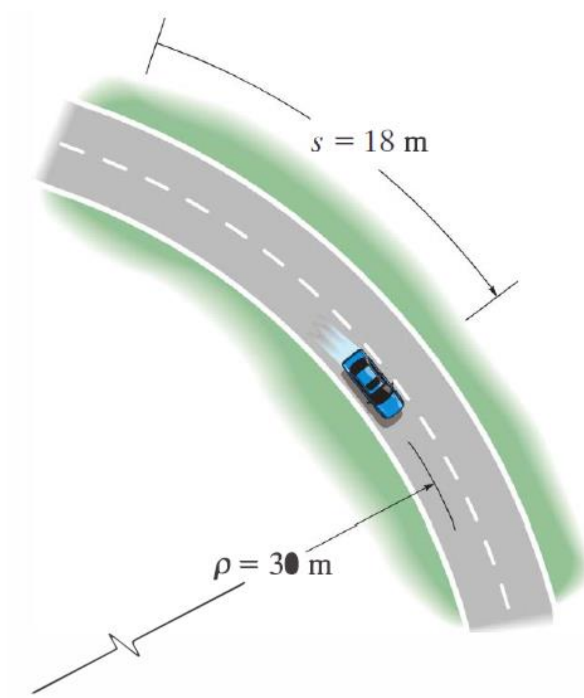
PROBLEMS :-

Q1/ When designing a highway curve it is required that cars traveling at a constant speed of 25 m/s must not have an acceleration that exceeds 3 m/s^2. Determine the Minimum radius of curvature of the curve?

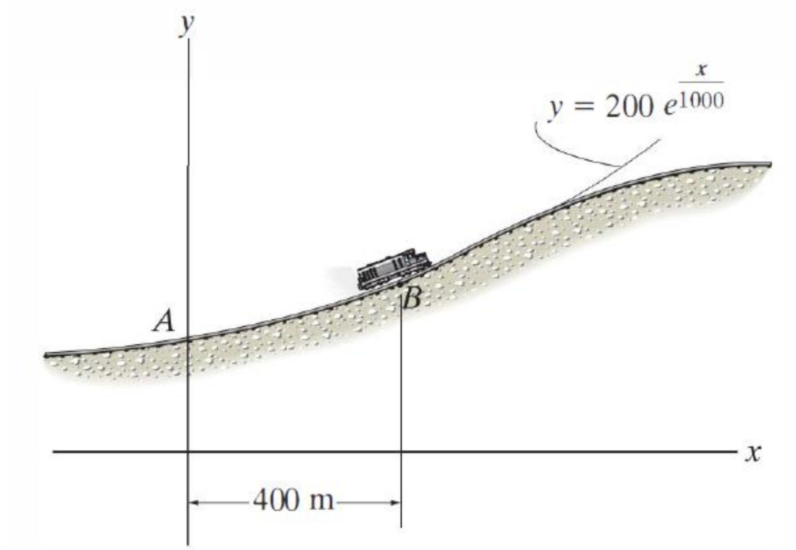
Q2/ Starting from rest, the motorboat travels around the circular path, $\rho = 50$ m, at a speed $v = (0.2t^2)$ m/s, where t is in seconds. Determine the magnitudes of the boat's velocity and acceleration at the instant t = 3 s.



Q3/ The car travels along the circular path such that its speed is increased by $a_t = (0.5e^t)$ m/s^2, where t is in seconds. Determine the magnitudes of its velocity and acceleration after the car has traveled s = 18 m starting from rest. Neglect the size of the car ?



Q4/ The train passes point A with a speed of 30 m/s and begins to decrease its speed at a constant rate of at = - 0.25 m/s². Determine the magnitude of the acceleration of the train when it reaches point B, where S_{AB} = 412 m?



Kinetics of a Particle
Force and Acceleration

Newton's Second Law of Motion:-

Kinetics is a branch of dynamics that deals with the relationship between the change in motion of a body and the forces that cause this change. The basis for kinetics is Newton's second law, which states that when an unbalanced force acts on a particle, the particle will accelerate in the direction of the force with a magnitude that is proportional to the force. This law can be verified experimentally by applying a known

Unbalanced force F to a particle, and then measuring the acceleration a. Since the force and acceleration are directly proportional, the constant of proportionality, m, may be determined from the ratio m = F / a. This positive scalar m is called the mass of the particle. Being constant during any acceleration, m provides a quantitative measure of the resistance of the particle to a change in its velocity that is its inertia. If the mass of the particle is m, Newton's second law of motion may be written in mathematical form as.

$$\mathbf{F} = m\mathbf{a}$$

Newton's Law of Gravitational Attraction. Shortly after formulating his three laws of motion, Newton postulated a law governing the mutual attraction between any two particles. In mathematical form this law can be expressed as.

$$F = G\frac{m_1 m_2}{r^2}$$

Where

F = force of attraction between the two particles

G = universal constant of gravitation; according to experimental evidence $G = 66.73(10^{-12})\ \mathrm{m^3/(kg \cdot s^2)}$

m_1, m_2 = mass of each of the two particles

r = distance between the centers of the two particles

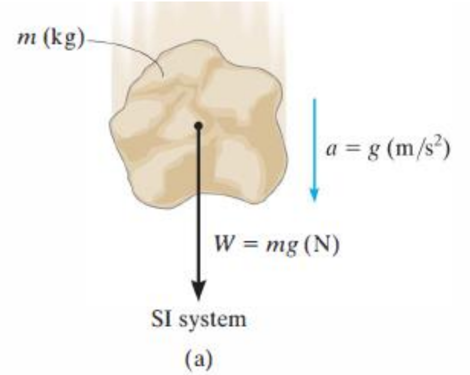
In the case of a particle located at or near the surface of the earth, the only gravitational force having any sizable magnitude is that between the earth and the particle. This force is termed the "weight" and, for our purpose, it will be the only gravitational force considered. From a above Eq, we can develop a general expression for finding the weight W of a particle having a mass $m_l = m$. Let $m_2 = M_e$ be the mass of the earth and r the distance between the earth's center and the particle. Then, if $g = GM_e/r^2$, we have

$$W = mg$$

By comparison with F = ma, we term g the acceleration due to gravity. For most engineering calculations g is a point on the surface of the earth at sea level, and at a latitude of 45°, which is considered the "standard location." Here the values $g = 9.81\ m/s^2 = 32.2\ ft/s^2$ will be used for calculations

$$W = mg \text{ (N)} \qquad (g = 9.81 \text{ m/s}^2) \qquad (13\text{–}2)$$

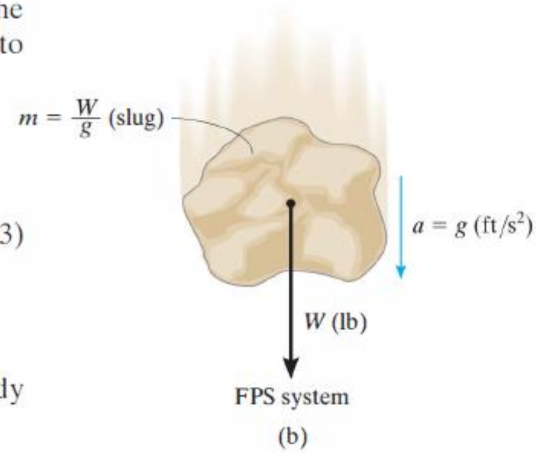
As a result, a body of mass 1 kg has a weight of 9.81 **N**; a 2-kg body weighs 19.62 **N**; and so on.



In the FPS system the weight of the body is specified in pounds. The mass is measured in slugs, a term derived from "sluggish" which refers to the body's inertia. It must be calculated, Fig. 13–1*b*, using

$$m = \frac{W}{g} \text{ (slug)} \qquad (g = 32.2 \text{ ft/s}^2) \qquad (13\text{–}3)$$

Therefore, a body weighing 32.2 lb has a mass of 1 slug; a 64.4-lb body has a mass of 2 slugs; and so on.



The Equation of Motion

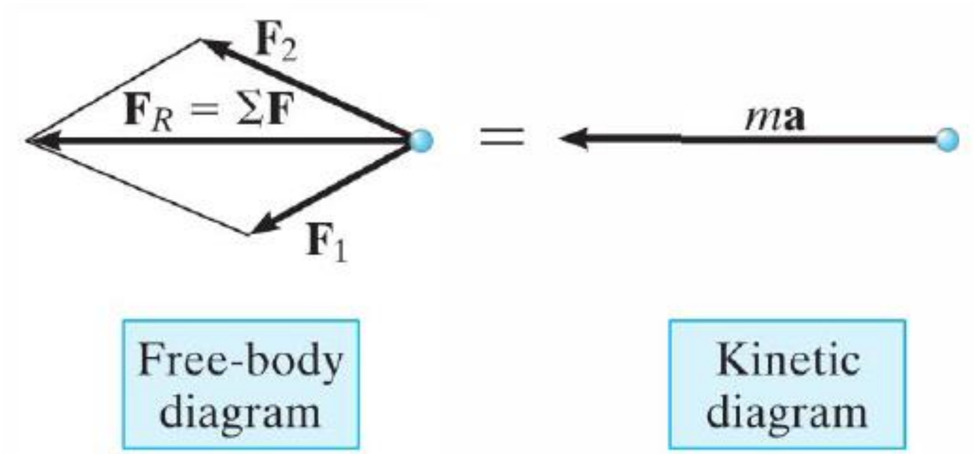
When more than one force acts on a particle, the resultant force is determined by a vector summation of all the forces; i.e., $F_R = \sum F$. For this more general case, the equation of motion may be written as

$$\sum \mathbf{F} = m\mathbf{a}$$

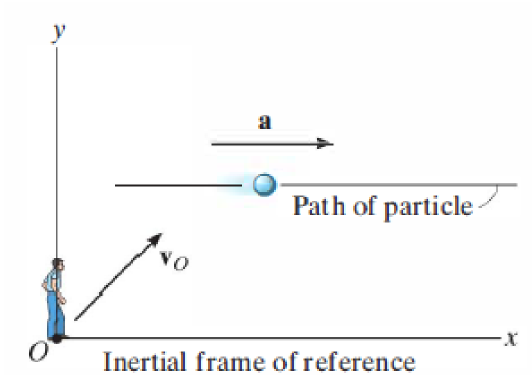
To illustrate application of this equation, consider the particle shown in Fig. a, which has a mass m and is subjected to the action of two forces, F_1 and F_2 • We can graphically account for the

(a)

magnitude and direction of each force acting on the particle by drawing the particle's free-body diagram, Fig b. Since the resultant of these forces produces the vector rna, its magnitude and direction can be represented graphically on the kinetic diagram, shown in Fig. c. The equal sign written between the diagrams symbolizes the graphical equivalency between the free-body diagram and the kinetic diagram; i.e., $\sum F = ma$.t In particular, note that if $F_R = \sum F = 0$, then the acceleration is also zero, so that the particle will either remain at rest or move along a estqruaiilgibhrt-iluinme, path with constant velocity. Such are the conditions of static Newton's first law of motion.



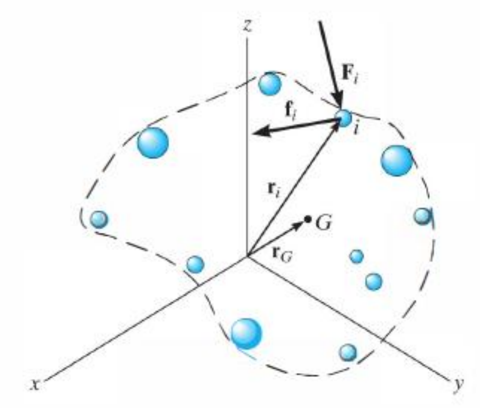
Inertial Reference Frame. When applying the equation of motion, it is important that the acceleration of the particle be measured with respect reference frame that either fixed or translate with a constant velocity In this way, the observer will not accelerate and measurements of the particle's acceleration will be the same from any reference of this type. Such a frame of reference is commonly known as a Newtonian or inertial reference frame



When studying the motions of rockets and satellites, it is justifiable to consider the inertial reference frame as fixed to the stars, whereas dynamics problems concerned with motions on or near the surface of the earth may be solved by using an inertial frame which is assumed fixed to the earth. Even though the earth both rotates about its own axis and revolves about the sun, the accelerations created by these rotations are relatively small and so they can be neglected for most applications.

Equation of Motion for a System of Particles

The equation of motion will now be extended to include a system of particles isolated within an enclosed region in space, as shown in Fig. a. In particular, there is no restriction in the way the particles are connected, so the following analysis applies equally well to the motion of a solid, liquid, or gas system.

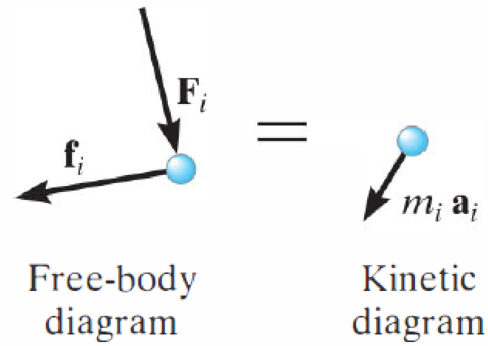


Inertial coordinate system
(a)

At the instant considered, the arbitrary *i*-th particle, having a mass m_i, is subjected to a system of internal forces and a resultant external force. The internal force, represented symbolically as f_i, is the resultant of all the forces the other particles exert on the *i* th particle. The resultant external force F_i represents, for example, the effect of gravitational, electrical, magnetic, or contact forces between the *i*th particle and adjacent bodies or particles not included within the system. The free-body and kinetic diagrams for the *i*th particle are shown in Fig. b. Applying the equation of motion,

$$\Sigma \mathbf{F} = m\mathbf{a};$$

$$\mathbf{F}_i + \mathbf{f}_i = m_i\mathbf{a}_i$$



(b)

When the equation of motion is applied to each of the other particles of the system, similar equations will result. And, if all these equations are added together vectorially, we obtain

$$\Sigma\mathbf{F}_i + \Sigma\mathbf{f}_i = \Sigma m_i\mathbf{a}_i$$

The summation of the internal forces, if carried out, will equal zero, since internal forces between any two particles occur in equal but opposite collinear pairs. Consequently, only the sum of the external forces will remain, and therefore the equation of motion, written for the system of particles, becomes

$$\Sigma\mathbf{F}_i = \Sigma m_i\mathbf{a}_i$$

If $\mathbf{r}_G$ is a position vector which locates the center of mass G of the particles, Fig. a, then by definition of the center of mass, m $\mathbf{r}_G = \sum .m_i r_i$, where $m = \sum m_i$ is the total mass of all the particles. Differentiating this equation twice with respect to time, assuming that no mass is entering or leaving the system, yields

$$m\mathbf{a}_G = \Sigma m_i\mathbf{a}_i$$

$$\Sigma\mathbf{F} = m\mathbf{a}_G$$

Rectilinear Motion:-

We now apply the concepts discussed in above Arts. of particle motion, starting with rectilinear motion in this article and treating curvilinear motion. In both articles, we will analyze the motions of bodies which can be treated as particles. This simplification is possible as long as we are interested only in the motion of the mass center of the body. In this case we may treat the forces as concurrent through the mass center.

If we choose the *x*-direction, for example, as the direction of the rectilinear motion of a particle of mass *m*, the acceleration in the *y*- and *z*-directions will be zero and the scalar components become

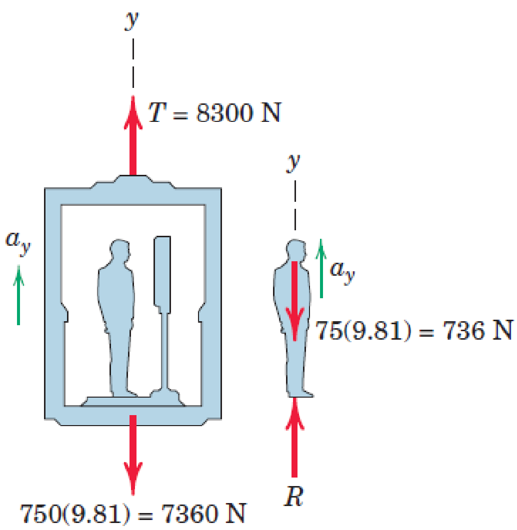
$$\Sigma F_x = ma_x$$
$$\Sigma F_y = 0$$
$$\Sigma F_z = 0$$

For cases where we are not free to choose a coordinate direction along the motion, we would have in the general case all three component equations.

$$\Sigma F_x = ma_x$$
$$\Sigma F_y = ma_y$$
$$\Sigma F_z = ma_z$$

where the acceleration and resultant force are given by

$$\mathbf{a} = a_x\mathbf{i} + a_y\mathbf{j} + a_z\mathbf{k}$$
$$a = \sqrt{a_x{}^2 + a_y{}^2 + a_z{}^2}$$
$$\Sigma\mathbf{F} = \Sigma F_x\mathbf{i} + \Sigma F_y\mathbf{j} + \Sigma F_z\mathbf{k}$$
$$|\Sigma\mathbf{F}| = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2 + (\Sigma F_z)^2}$$



EXA M P L E-1 -

A 75-kg man stands on a spring scale in an elevator. During the first 3 seconds of motion from rest, the tension T in the hoisting cable is 8300 N. Find the reading R of the scale in newtons during this interval and the upward velocity v of the elevator at the end of the 3 seconds. The total mass of the elevator, man, and scale is 750 kg

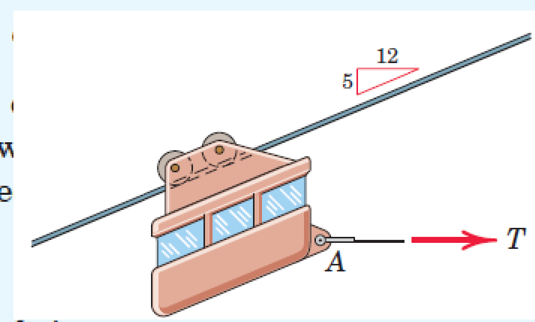
.

Solution. The force registered by the scale and the velocity both depend on the acceleration of the elevator, which is constant during the interval for which the forces are constant. From the free-body diagram of the elevator, scale, and man taken together, the acceleration is found to be

$[\Sigma F_y = ma_y] \qquad 8300 - 7360 = 750a_y$

The scale reads the downward force exerted ... l
and opposite reaction R to this action is show... e
man alone together with his weight, and the e...

$[\Sigma F_y = ma_y] \qquad R - 736 = 75(1.257)$ ... .



The velocity reached at the end of the 3 seconds is

$$[\Delta v = \int a\, dt] \qquad v - 0 = \int_0^3 1.257\, dt \qquad v = 3.77 \text{ m/s} \qquad \textit{Ans.}$$

EXA M P L E-2 –

A small inspection car with a mass of 200 kg runs along the fixed overhead cable and is controlled by the attached cable at *A*. Determine the acceleration of the car when the control cable is horizontal and under a tension $T = 2.4$ kN. Also find the total force *P* exerted by the supporting cable on the wheels.

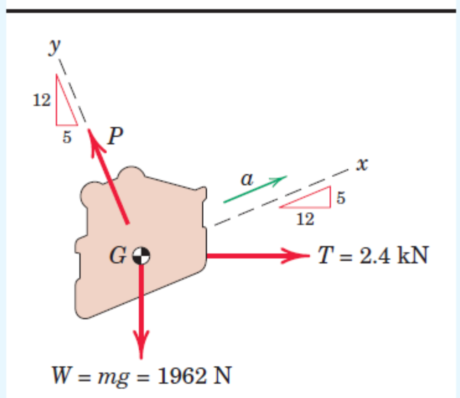
Solution. The free-body diagram of the car and wheels taken together and treated as a particle discloses the 2.4-kN tension T, the weight $W = mg = 200(9.81) = 1962$ N, and the force P exerted on the wheel assembly by the cable.

The car is in equilibrium in the y-direction since there is no acceleration in this direction. Thus,

$[\Sigma F_y = 0]$ $\qquad P - 2.4(\frac{5}{13}) - 1.962(\frac{12}{13}) = 0 \qquad P = 2.73 \text{ kN}$ *Ans.*

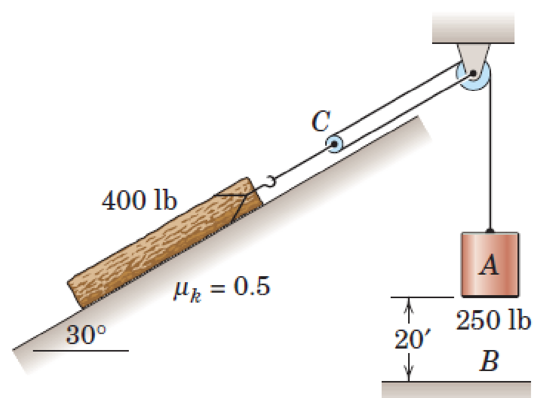
In the x-direction the equation of motion gives

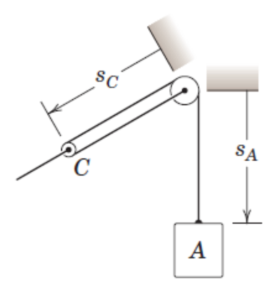
$[\Sigma F_x = ma_x]$ $\qquad 2400(\frac{12}{13}) - 1962(\frac{5}{13}) = 200a \qquad a = 7.30 \text{ m/s}^2$ *Ans.*

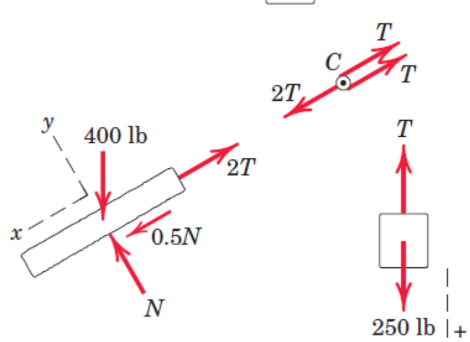


EXAMPLES -3-

The 250-lb concrete block *A* is released from rest in the position shown and pulls the 400-lb log up the 30_ ramp. If the coefficient of kinetic friction between the log and the ramp is 0.5, determine the velocity of the block as it hits the ground at *B*.







Solution. The motions of the log and the block A are clearly dependent. Although by now it should be evident that the acceleration of the log up the incline is half the downward acceleration of A, we may prove it formally. The constant total length of the cable is $L = 2s_C + s_A$ + constant, where the constant accounts for the cable portions wrapped around the pulleys. Differentiating twice with respect to time gives $0 = 2\ddot{s}_C + \ddot{s}_A$, or

$$0 = 2a_C + a_A$$

We assume here that the masses of the pulleys are negligible and that they turn with negligible friction. With these assumptions the free-body diagram of the pulley C discloses force and moment equilibrium. Thus, the tension in the cable attached to the log is twice that applied to the block. Note that the accelerations of the log and the center of pulley C are identical.

The free-body diagram of the log shows the friction force $\mu_k N$ for motion up the plane. Equilibrium of the log in the y-direction gives

$$[\Sigma F_y = 0] \qquad N - 400 \cos 30° = 0 \qquad N = 346 \text{ lb}$$

and its equation of motion in the x-direction gives

$$[\Sigma F_x = ma_x] \qquad 0.5(346) - 2T + 400 \sin 30° = \frac{400}{32.2} a_C$$

For the block in the positive downward direction, we have

$$[+\downarrow \Sigma F = ma] \qquad 250 - T = \frac{250}{32.2} a_A$$

Solving the three equations in a_C, a_A, and T gives us

$$a_A = 5.83 \text{ ft/sec}^2 \qquad a_C = -2.92 \text{ ft/sec}^2 \qquad T = 205 \text{ lb}$$

For the 20-ft drop with constant acceleration, the block acquires a velocity

$$[v^2 = 2ax] \qquad v_A = \sqrt{2(5.83)(20)} = 15.27 \text{ ft/sec} \qquad \textit{Ans.}$$

EXAMPLE -4

A 10-kg projectile is fired vertically upward from the ground, with an initial velocity of 50 m/s, Fig. 13–7*a*. Determine the maximum height to which it will travel if (*a*) atmospheric resistance is neglected; and (*b*) atmospheric resistance is measured as $F_D = (0.01v^2)$ N, where v is the speed of the projectile at any instant, measured in m/s.

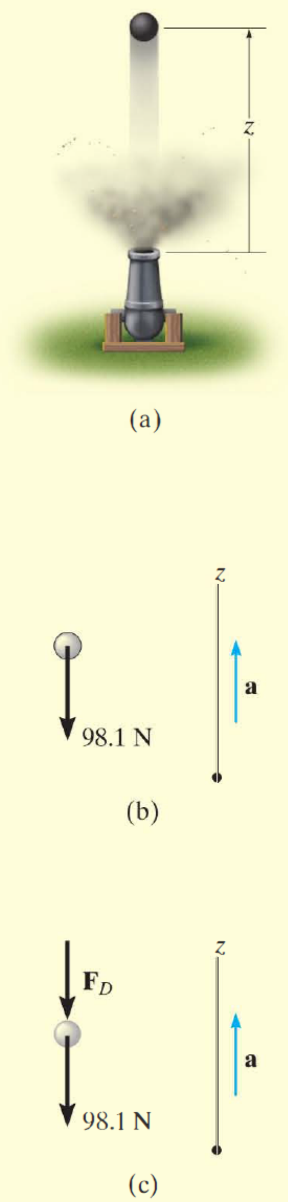


Fig. 13–7

SOLUTION

In both cases the known force on the projectile can be related to its acceleration using the equation of motion. Kinematics can then be used to relate the projectile's acceleration to its position.

Part (a) Free-Body Diagram. As shown in Fig. 13–7*b*, the projectile's weight is $W = mg = 10(9.81) = 98.1$ N. We will assume the unknown acceleration **a** acts upward in the *positive z* direction.

Equation of Motion.

$$+\uparrow \Sigma F_z = ma_z; \qquad -98.1 = 10a, \qquad a = -9.81 \text{ m/s}^2$$

The result indicates that the projectile, like every object having free-flight motion near the earth's surface, is subjected to a *constant* downward acceleration of 9.81 m/s^2.

Kinematics. Initially, $z_0 = 0$ and $v_0 = 50$ m/s, and at the maximum height $z = h$, $v = 0$. Since the acceleration is *constant*, then

$$(+\uparrow) \qquad v^2 = v_0^2 + 2a_c(z - z_0)$$

$$0 = (50)^2 + 2(-9.81)(h - 0)$$

$$h = 127 \text{ m} \qquad \textit{Ans.}$$

Part (b) Free-Body Diagram. Since the force $F_D = (0.01v^2)$ N tends to retard the upward motion of the projectile, it acts downward as shown on the free-body diagram, Fig. 13–7*c*.

Equation of Motion.

$$+\uparrow \Sigma F_z = ma_z; \qquad -0.01v^2 - 98.1 = 10a, \qquad a = -(0.001v^2 + 9.81)$$

Kinematics. Here the acceleration is *not constant* since F_D depends on the velocity. Since $a = f(v)$, we can relate a to position using

$$(+\uparrow)\ a\,dz = v\,dv; \qquad -(0.001v^2 + 9.81)\,dz = v\,dv$$

Separating the variables and integrating, realizing that initially $z_0 = 0$, $v_0 = 50$ m/s (positive upward), and at $z = h$, $v = 0$, we have

$$\int_0^h dz = -\int_{50}^0 \frac{v\,dv}{0.001v^2 + 9.81} = -500\ln(v^2 + 9810)\Big|_{50\text{ m/s}}^{0}$$

$$h = 114 \text{ m} \qquad \textit{Ans.}$$

EXAMPLE -5-

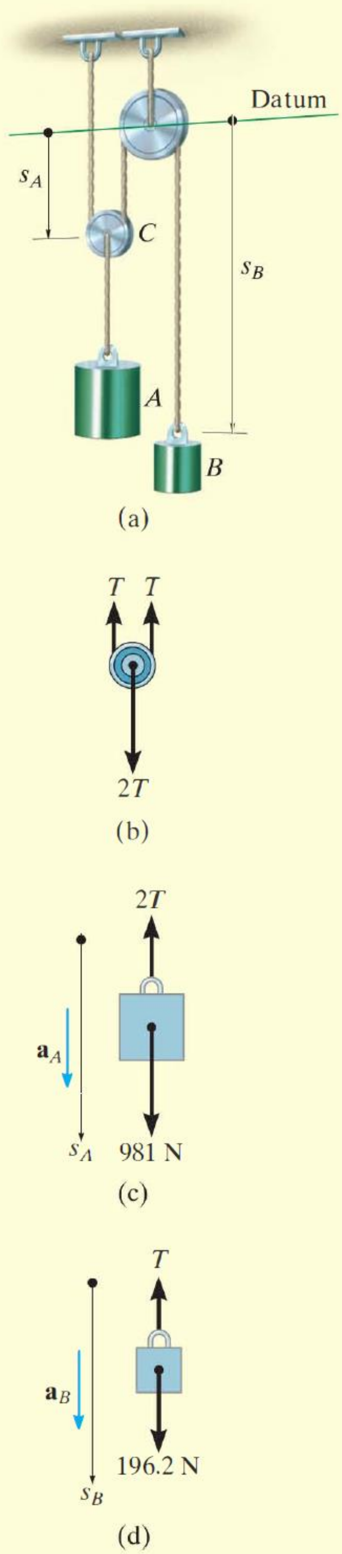


Fig. 13–10

The 100-kg block A shown in Fig. 13–10a is released from rest. If the masses of the pulleys and the cord are neglected, determine the speed of the 20-kg block B in 2 s.

SOLUTION

Free-Body Diagrams. Since the mass of the pulleys is *neglected*, then for pulley C, $ma = 0$ and we can apply $\Sigma F_y = 0$ as shown in Fig. 13–10b. The free-body diagrams for blocks A and B are shown in Fig. 13–10c and d, respectively. Notice that for A to remain stationary $T = 490.5$ N, whereas for B to remain static $T = 196.2$ N. Hence A will move down while B moves up. Although this is the case, we will assume both blocks accelerate downward, in the direction of $+s_A$ and $+s_B$. The three unknowns are T, a_A, and a_B.

Equations of Motion. Block A,

$$+\downarrow \Sigma F_y = ma_y; \qquad 981 - 2T = 100a_A \qquad (1)$$

Block B,

$$+\downarrow \Sigma F_y = ma_y; \qquad 196.2 - T = 20a_B \qquad (2)$$

Kinematics. The necessary third equation is obtained by relating a_A to a_B using a dependent motion analysis, discussed in Sect. 12.9. The coordinates s_A and s_B in Fig. 13–10a measure the positions of A and B from the fixed datum. It is seen that

$$2s_A + s_B = l$$

where l is constant and represents the total vertical length of cord. Differentiating this expression twice with respect to time yields

$$2a_A = -a_B \qquad (3)$$

Notice that when writing Eqs. 1 to 3, the *positive direction was always assumed downward*. It is very important to be *consistent* in this assumption since we are seeking a simultaneous solution of equations. The results are

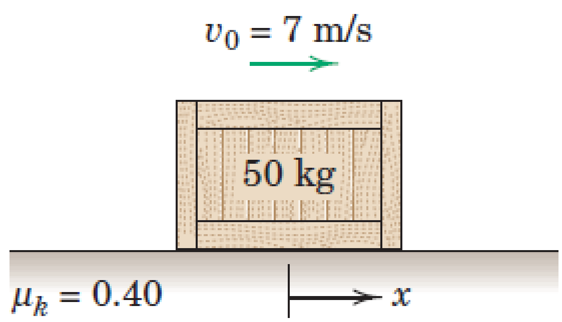
$$T = 327.0 \text{ N}$$
$$a_A = 3.27 \text{ m/s}^2$$
$$a_B = -6.54 \text{ m/s}^2$$

Hence when block A accelerates *downward*, block B accelerates *upward* as expected. Since a_B is constant, the velocity of block B in 2 s is thus

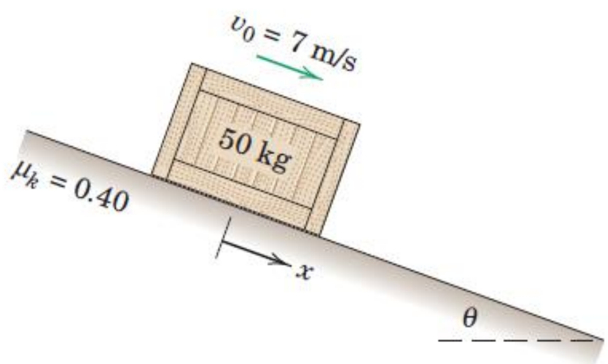
$$(+\downarrow) \qquad v = v_0 + a_B t$$
$$= 0 + (-6.54)(2)$$
$$= -13.1 \text{ m/s} \qquad \textit{Ans.}$$

PROBLEMS

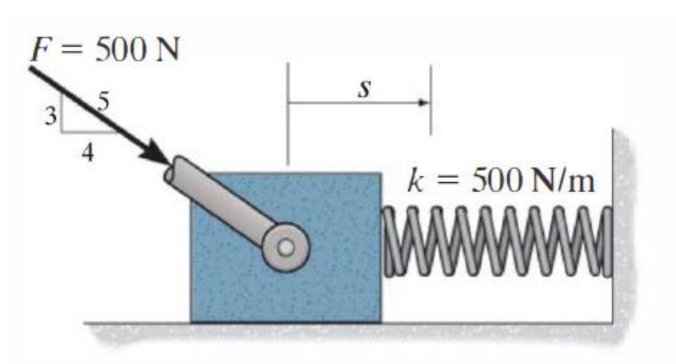
Q1 /The 50-kg crate is projected along the floor with an initial speed of 7 m/s at x=0 . The coefficient of kinetic friction is 0.40. Calculate the time required for the crate to come to rest and the corresponding distance x traveled



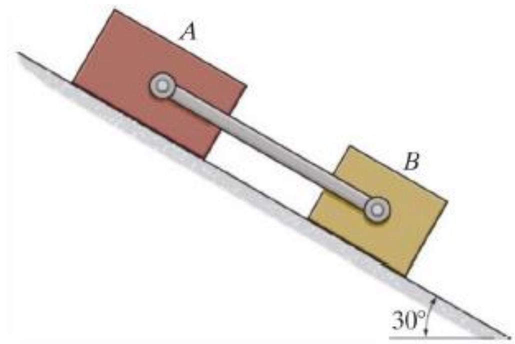
Q2/ The 50-kg crate of Prob. 1 is now projected down an incline as shown with an initial speed of 7 m/s. Investigate the time t required for the crate to come to rest and the corresponding distance x traveled if (*a*) θ=15° and (*b*) θ=30° .



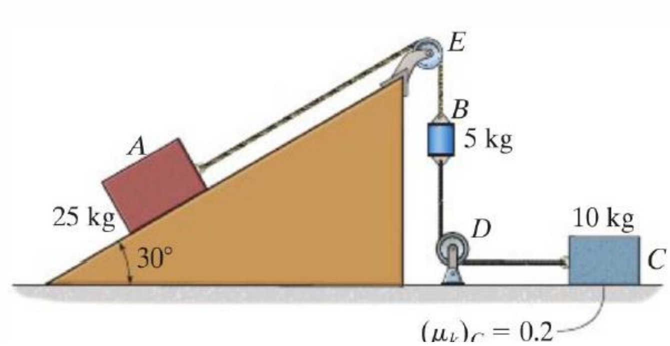
Q3/ A spring of stiffness k = 500 N/m is mounted against the 10-kg block. If the block is subjected to the force of F = 500 N, determine its velocity at s = 0.5 m. When s = 0, the block is at rest and the spring is uncompressed. The contact surface is smooth.



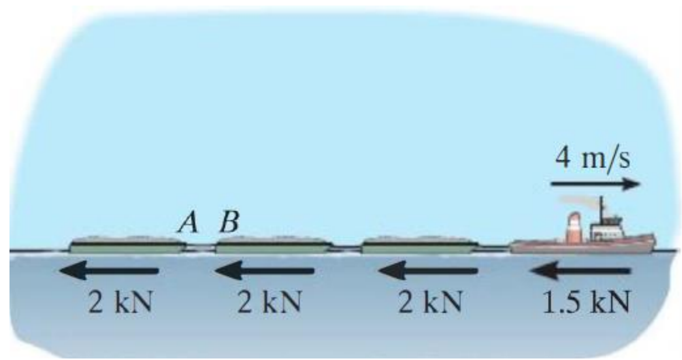
Q4/ If block A and B of mass 10 kg ans 6 kg respectively, are placed on the inclined plane and released, determine the force developed in the link. The coefficients of kinetic friction between the blocks and the inclined plane are $\mu_A = 0.1$ and $\mu_B = 0.3$. Neglect the mass of the link.



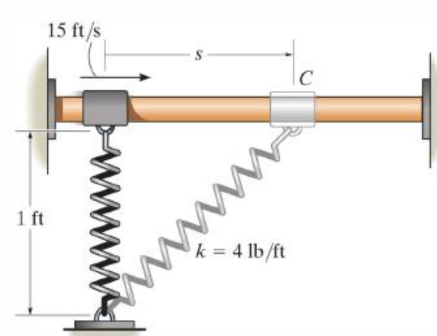
Q5/ Determine the acceleration of the system and the tension in each cable. The inclined plane is smooth, and the coefficient of kinetic friction between the horizontal surface and block C is $(\mu k)_c = 0.2$.



Q6/Each of the three barges has a mass of 30 Mg, whereas the tugboat has a mass of 12 Mg. As the barges are being pulled forward with a constant velocity of 4 m/s, the tugboat must overcome the frictional resistance of the water, which is 2 kN for each barge and 1.5 kN for the tugboat. If the cable between A and B breaks, determine the acceleration of the tugboat

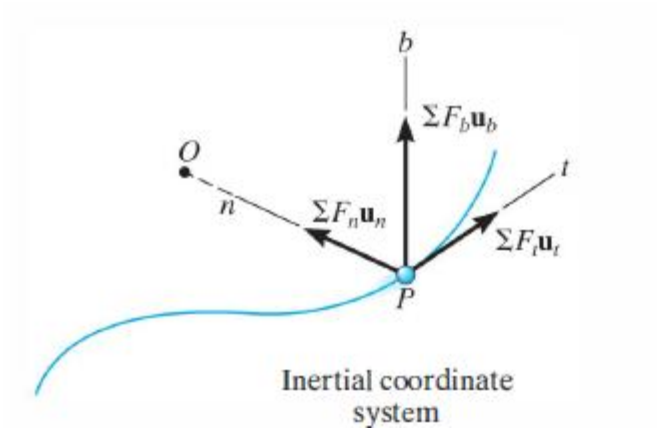


Q7/ The 2-lb collar C fits loosely on the smooth shaft. If the spring is unstretched when s = 0 and the collar is given a velocity of 1 5 ft/s, determine the velocity of the collar when $s = 1$ ft.



Equations of Motion Normal and Tangential Coordinates

When a particle moves along a curved path which is known, the equation of motion for the particle may be written in the tangential, normal, and bi normal directions, Fig.. Note that there is no motion of the particle in the bi normal direction, since the particle is constrained to move along the path. We have



This equation is satisfied provided

$$\Sigma F_t = ma_t$$
$$\Sigma F_n = ma_n$$
$$\Sigma F_b = 0$$

Recall that a_t (= dv/ dt) represents the time rate of change in the magnitude of velocity. So if $\sum F_t$ acts in the direction of motion, the particle's speed will increase, whereas if it acts in the opposite direction, the particle will slow down. Likewise, a_n (= v^2 / ρ) represents the time rate of change in the velocity's direction. It is caused by $\sum F_n$, which always acts in the positive n direction, i.e., toward the path's center of curvature. From this reason it is often referred to as the centripetal force

Example -1-

Determine the banking angle θ for the race track so that the wheels of the racing cars shown in Fig. a will not have to depend upon friction to prevent any car from sliding up or down the track. Assume the cars have negligible size, a mass m, and travel around the curve of radius ρ with a constant speed υ.


(a)

SOLUTION

Before looking at the following solution, give some thought as to why it should be solved using t, n, b coordinates.

Free-Body Diagram. As shown in Fig. 13–12*b*, and as stated in the problem, no frictional force acts on the car. Here $\mathbf{N}_C$ represents the *resultant* of the ground on all four wheels. Since a_n can be calculated, the unknowns are N_C and θ.

Equations of Motion. Using the n, b axes shown,

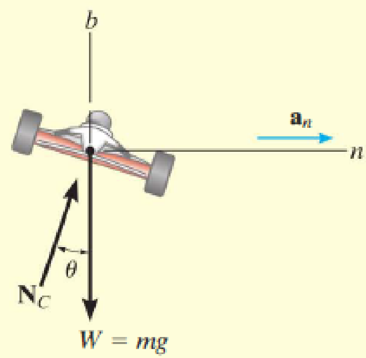
$$\overset{+}{\rightarrow} \Sigma F_n = ma_n; \qquad N_C \sin\theta = m\frac{v^2}{\rho} \qquad (1)$$

$$+\uparrow \Sigma F_b = 0; \qquad N_C \cos\theta - mg = 0 \qquad (2)$$

Eliminating N_C and m from these equations by dividing Eq. 1 by Eq. 2, we obtain

$$\tan\theta = \frac{v^2}{g\rho}$$

$$\theta = \tan^{-1}\left(\frac{v^2}{g\rho}\right) \qquad \textit{Ans.}$$

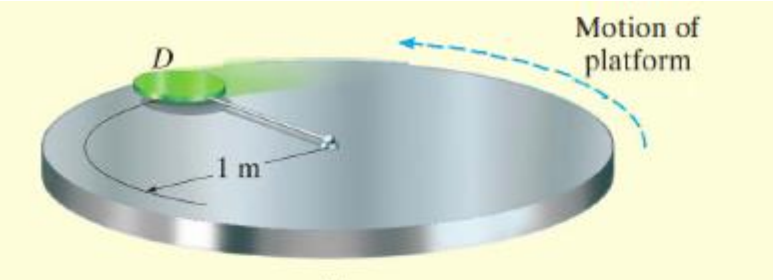


(b)

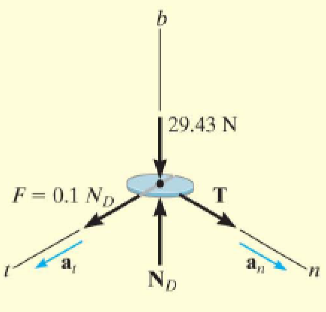
Fig. 13–12

Example -2-

The 3-kg disk D is attached to the end of a cord as shown in Fig. a. The other end of the cord is attached to a ball-and-socket joint located at the center of a platform. If the platform rotates rapidly, and the disk is placed on it and released from rest as shown, determine the time it takes for the disk to reach a speed great enough to break the cord. The maximum tension the cord can sustain is 100 N, and the coefficient of kinetic friction between the disk and the platform is $\mu_k = 0.1$.



(a)



(b)

Fig. 13–13

SOLUTION

Free-Body Diagram. The frictional force has a magnitude $F = \mu_k N_D = 0.1N_D$ and a sense of direction that opposes the *relative motion* of the disk with respect to the platform. It is this force that gives the disk a tangential component of acceleration causing v to increase, thereby causing T to increase until it reaches 100 N. The weight of the disk is $W = 3(9.81) = 29.43$ N. Since a_n can be related to v, the unknowns are N_D, a_t, and v.

Equations of Motion.

$$\Sigma F_n = ma_n; \qquad T = 3\left(\frac{v^2}{1}\right) \qquad (1)$$

$$\Sigma F_t = ma_t; \qquad 0.1N_D = 3a_t \qquad (2)$$

$$\Sigma F_b = 0; \qquad N_D - 29.43 = 0 \qquad (3)$$

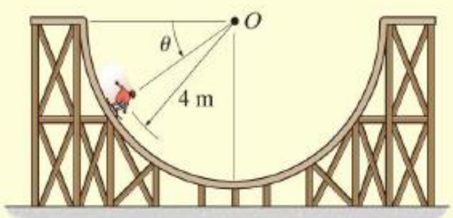
Setting $T = 100$ N, Eq. 1 can be solved for the critical speed v_{cr} of the disk needed to break the cord. Solving all the equations, we obtain

$$N_D = 29.43 \text{ N}$$
$$a_t = 0.981 \text{ m/s}^2$$
$$v_{cr} = 5.77 \text{ m/s}$$

Kinematics. Since a_t is *constant*, the time needed to break the cord is

$$v_{cr} = v_0 + a_t t$$
$$5.77 = 0 + (0.981)t$$
$$t = 5.89 \text{ s} \qquad \textit{Ans.}$$

Example -3-

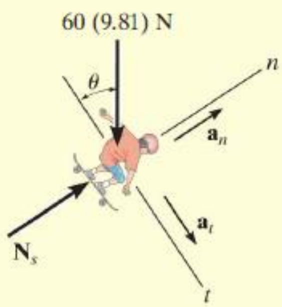


(a)

The 60-kg skateboarder in Fig.13–15*a* coasts down the circular track. If he starts from rest when $\theta = 0°$, determine the magnitude of the normal reaction the track exerts on him when $\theta = 60°$. Neglect his size for the calculation.

SOLUTION

Free-Body Diagram. The free-body diagram of the skateboarder when he is at an *arbitrary position* θ is shown in Fig. 13–15*b*. At $\theta = 60°$ there are three unknowns, N_s, a_t, and a_n (or v).



(b)

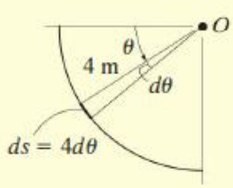
Equations of Motion.

$$\downarrow \Sigma F_n = ma_n;\quad N_S - [60(9.81)\text{N}]\sin\theta = (60\text{ kg})\left(\frac{v^2}{4\text{m}}\right) \qquad (1)$$

$$\downarrow \Sigma F_t = ma_t;\quad [60(9.81)\text{N}]\cos\theta = (60\text{ kg})\, a_t$$

$$a_t = 9.81\cos\theta$$

Kinematics. Since a_t is expressed in terms of θ, the equation $v\,dv = a_t\,ds$ must be used to determine the speed of the skateboarder when $\theta = 60°$. Using the geometric relation $s = \theta r$, where $ds = r\,d\theta = (4\text{ m})\,d\theta$, Fig. 13–15*c*, and the initial condition $v = 0$ at $\theta = 0°$, we have,



(c)

Fig. 13–15

$$v\,dv = a_t\,ds$$

$$\int_0^v v\,dv = \int_0^{60°} 9.81\cos\theta(4\,d\theta)$$

$$\left.\frac{v^2}{2}\right|_0^v = 39.24\sin\theta\Big|_0^{60°}$$

$$\frac{v^2}{2} - 0 = 39.24(\sin 60° - 0)$$

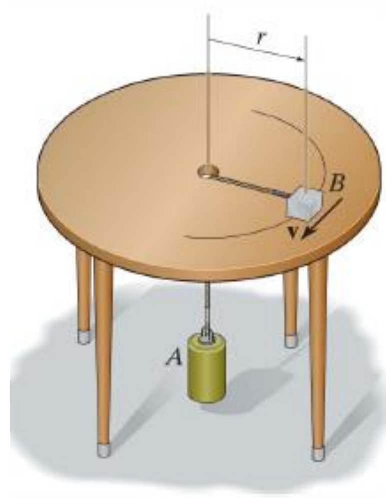
$$v^2 = 67.97\text{ m}^2/\text{s}^2$$

Substituting this result and $\theta = 60°$ into Eq. (1), yields

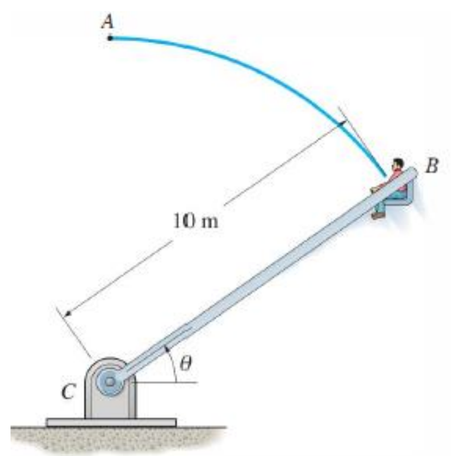
$$N_s = 1529.23\text{ N} = 1.53\text{ kN} \qquad \textit{Ans.}$$

Problems

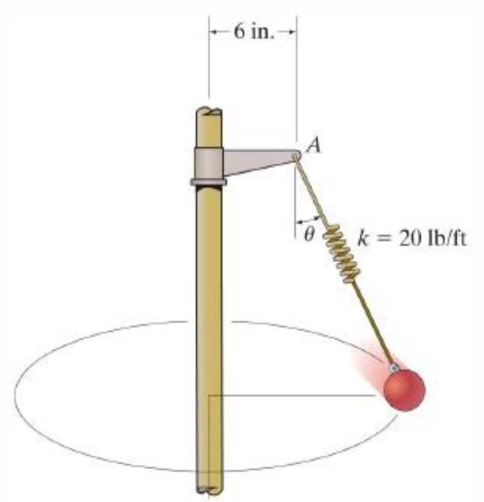
Q1/ The 2-kg block B and 15-kg cylinder A are connected to a light cord that passes through a hole in the center of the smooth table. If the block travels along a circular path of radius r = 1.5 m, determine the speed of the block.



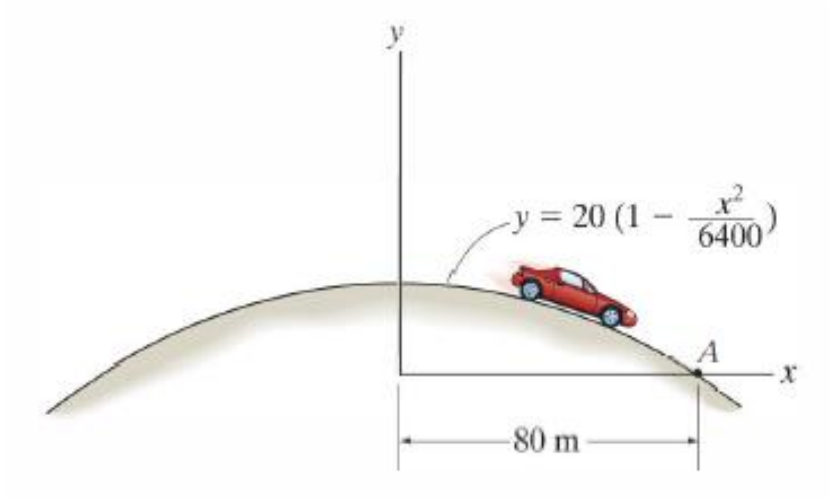
Q2/ A man having the mass of 75 kg sits in the chair which is pin-connected to the frame BC. If the man is always seated in an upright position, determine the horizontal and vertical reactions of the chair on the man at the instant θ= 45°. At this instant he has a speed of 6 m/s, which is increasing at 0.5 m/s^2



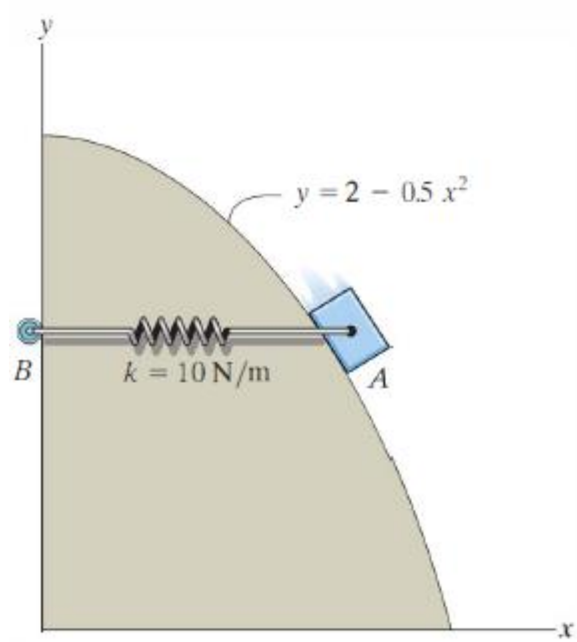
Q3/ A spring, having an unstretched length of 2 ft, has one end attached to the 10-lb ball. Determine the angle θ of the spring if the ball has a speed of 6 ft/s tangent to the horizontal circular path



Q4/ The 0.8-Mg car travels over the hill having the shape of a parabola. When the car is at point A, it is traveling at 9 m/s and increasing its speed at 3 m/s^2. Determine both the resultant normal force and the resultant frictional force that all the wheels of the car exert on the road at this instant. Neglect the size of the car

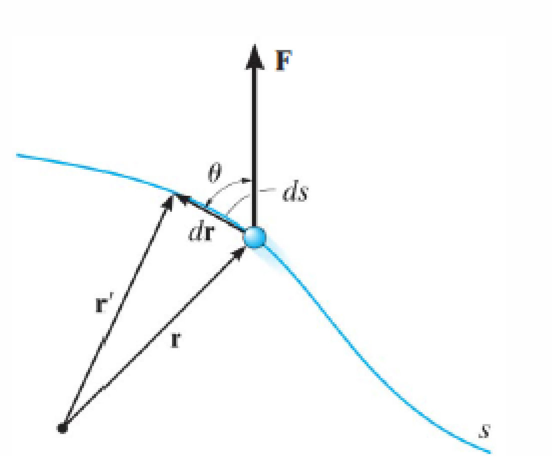


Q5/ The 6-kg block is confined to move along the smooth parabolic path. The attached spring restricts the motion and, due to the roller guide, always remains horizontal as the block descends. If the spring has a stiffness of k = 10 N/m, and unstretched length of 0.5 m, determine the normal force of the path on the block at the instant x = 1 m when the block has a speed of 4 m/s. Also, what is the rate of increase in speed of the block at this point? Neglect the mass of the roller and the spring.



Kinetics of a Particle
Work and Energy

The analyze motion of a particle using the concepts of work and energy. The resulting equation will be useful for solving problems that involve force, velocity, and displacement. Before we do this, however, we must first define the work of a force. Specifically, a force F will do work on a particle only when the particle undergoes a displacement in the direction of the force. For example, if the force F in Fig. causes the particle to move along the path s from position r to a new position r', the displacement is then dr = r' - r. The magnitude of dr is ds, the length of the differential segment along the path. If the angle between the tails of dr and F is e, Fig. then the work done by F is a scalar quantity, defined by



$$dU = F\,ds\cos\theta$$

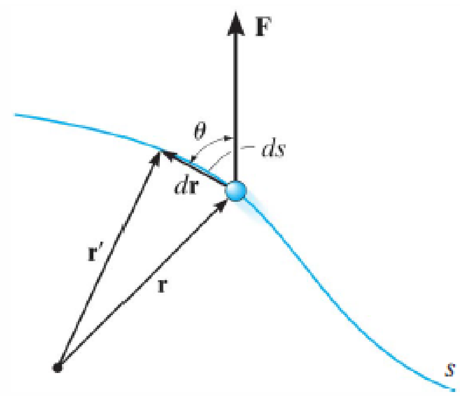
By definition of the dot product this equation can also be written as

$$dU = \mathbf{F}\cdot d\mathbf{r}$$

This result may be interpreted in one of two ways: either as the product of F and the component of displacement ds cosθ in the direction of the force, or as the product of ds and the component of force,

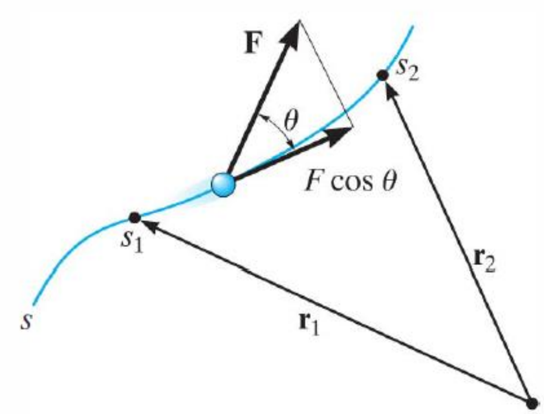
$F\cos\theta$, in the direction of displacement. Note that if $0^\circ \leq \theta < 90^\circ$, then the force component and the displacement have the *same sense* so that the work is *positive;* whereas if $90^\circ < \theta \leq 180^\circ$, these vectors will have *opposite sense*, and therefore the work is *negative*. Also, $dU = 0$ if the force is *perpendicular* to displacement, since $\cos 90^\circ = 0$, or if the force is applied at a *fixed point*, in which case the displacement is zero.

The unit of work in SI units is the joule (J), which is the amount of work done by a one-newton force when it moves through a distance of one meter in the direction of the force ($1\text{ J} = 1\text{ N}\cdot\text{m}$). In the FPS system, work is measured in units of foot-pounds (ft · lb), which is the work done by a one-pound force acting through a distance of one foot in the direction of the force.*



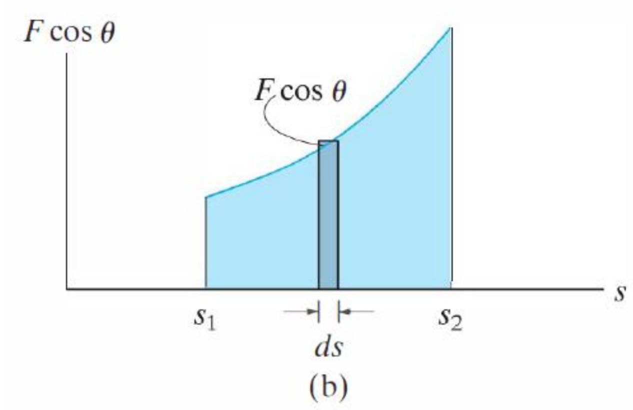
Work of a Varia ble Force.

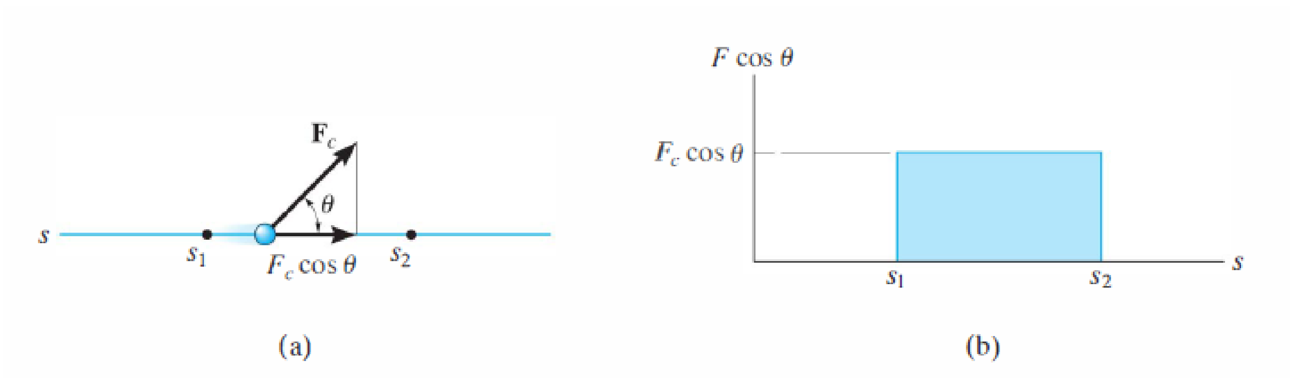
If the particle acted upon by the force F undergoes a finite displacement along its path from r_1 to r_2 or S_1 to S_2 , Fig.a, the work of force F is determined by integration. Provided F and 8 can be expressed as a function of position, then



$$U_{1-2} = \int_{\mathbf{r}_1}^{\mathbf{r}_2} \mathbf{F}\cdot d\mathbf{r} = \int_{s_1}^{s_2} F\cos\theta\, ds$$

Sometimes, this relation may be obtained by using experimental data to plot a graph of F cosθ vs. s. Then the area under this graph bounded by S_1 and S_2 represents the total work, Fig.b.





Work of a Consta nt Force Moving Along a Straight Line.

If the force F_c has a constant magnitude and acts at a constant angle θ from its straight-line path, Fig. a, then the component of F_c in the direction of displacement is always F_c cosθ. The work done by F_c when the particle is displaced from S_l to S_2 is determined from Eq. in which case

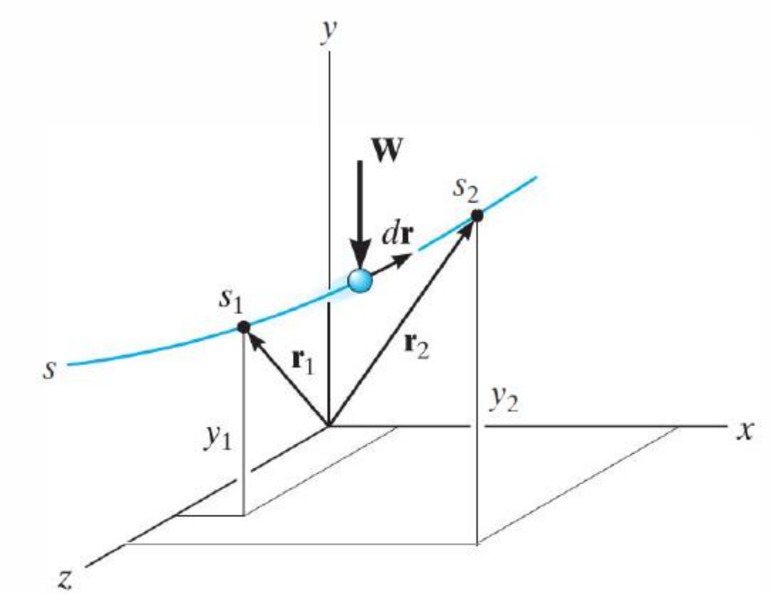
$$U_{1-2} = F_c \cos\theta \int_{s_1}^{s_2} ds$$

$$U_{1-2} = F_c \cos\theta (s_2 - s_1)$$

Work of a Weight.

Consider a particle of weight **W**, which moves up along the path s shown in Fig. from position S_1 to position S_2. At an intermediate point, the displacement

dr = dxi + dyj + dzk. Since W = -Wj, we have



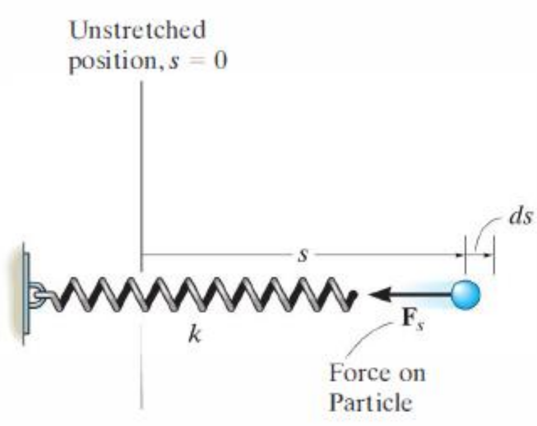
$$U_{1-2} = \int \mathbf{F} \cdot d\mathbf{r} = \int_{\mathbf{r}_1}^{\mathbf{r}_2} (-W\mathbf{j}) \cdot (dx\mathbf{i} + dy\mathbf{j} + dz\mathbf{k})$$

$$= \int_{y_1}^{y_2} -W\,dy = -W(y_2 - y_1)$$

$$\boxed{U_{1-2} = -W\,\Delta y}$$

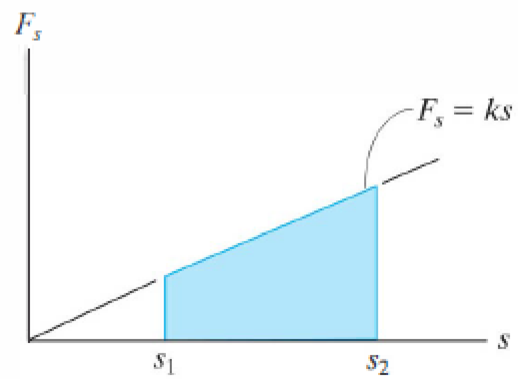
Work of a S pring Force.

If an elastic spring is elongated a distance ds, Fig. a, then the work done by the force that acts on the attached particle is dU = - $F_s$ds = -k_s ds. The work is negative since F_s acts in the opposite sense to ds. If the particle displaces from S_1 to S_2, the work of F_s is then



(a)

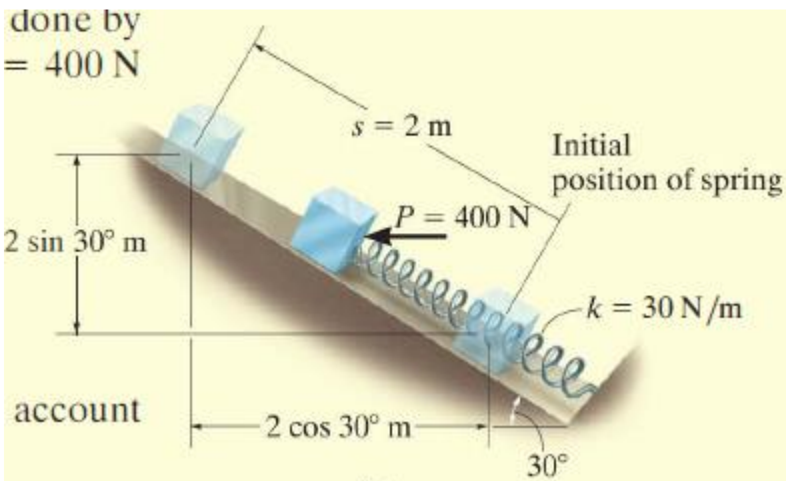
This work represents the trapezoidal area under the line $F_s = ks$, Fig.b.



(b)

Examples -1-

The l0-kg block shown in Fig. 14-6a rests on the smooth incline. If the spring is originally stretched 0.5 m, determine the total work done by all the forces acting on the block when a horizontal force P = 400 N pushes the block up the plane s = 2 m.



(a)

SOLUTION

First the free-body diagram of the block is drawn in order to account for all the forces that act on the block, Fig. 14–6*b*.

Horizontal Force *P*. Since this force is *constant*, the work is determined using Eq. 14–2. The result can be calculated as the force times the component of displacement in the direction of the force; i.e.,

$$U_P = 400\ \text{N}\ (2\ \text{m}\cos 30^\circ) = 692.8\ \text{J}$$

or the displacement times the component of force in the direction of displacement, i.e.,

$$U_P = 400\ \text{N}\cos 30^\circ(2\ \text{m}) = 692.8\ \text{J}$$

Spring Force F_s. In the initial position the spring is stretched $s_1 = 0.5$ m and in the final position it is stretched $s_2 = 0.5\ \text{m} + 2\ \text{m} = 2.5$ m. We require the work to be negative since the force and displacement are opposite to each other. The work of $\mathbf{F}_s$ is thus

$$U_s = -\left[\tfrac{1}{2}(30\ \text{N/m})(2.5\ \text{m})^2 - \tfrac{1}{2}(30\ \text{N/m})(0.5\ \text{m})^2\right] = -90\ \text{J}$$

Weight W. Since the weight acts in the opposite sense to its vertical displacement, the work is negative; i.e.,

$$U_W = -(98.1\ \text{N})\ (2\ \text{m}\sin 30^\circ) = -98.1\ \text{J}$$

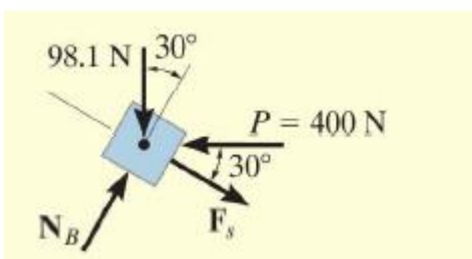
Note that it is also possible to consider the component of weight in the direction of displacement; i.e.,

$$U_W = -(98.1\sin 30^\circ\ \text{N})\ (2\ \text{m}) = -98.1\ \text{J}$$

Normal Force N_B. This force does *no work* since it is *always* perpendicular to the displacement.

Total Work. The work of all the forces when the block is displaced 2 m is therefore

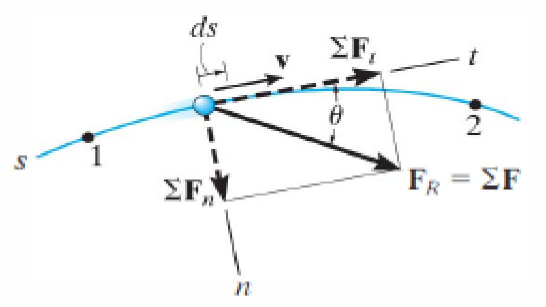
$$U_T = 692.8\ \text{J} - 90\ \text{J} - 98.1\ \text{J} = 505\ \text{J} \qquad \textit{Ans.}$$



(b)

Principle of Work and Energy

Consider the particle in Fig. which is located on the path defined relative to an inertial coordinate system. If the particle has a mass m and is subjected to a system of external forces represented by the resultant $F_R = \sum F$, then the equation of motion for the particle in the tangential direction is $\sum F_t = ma_t$. Applying the kinematic equation $a_t = v\, dv/ds$ and integrating both sides, assuming initially that the particle has a position $S = S_1$ and a speed $V = V_1$, and later at $S = S_2$, $V = V_2$, we have



$$\sum \int_{s_1}^{s_2} F_t\, ds = \int_{v_1}^{v_2} mv\, dv$$

$$\sum \int_{s_1}^{s_2} F_t\, ds = \tfrac{1}{2}mv_2^2 - \tfrac{1}{2}mv_1^2$$

From this Fig. note that $\sum Ft = \sum F\cos\theta$, and since work is defined from Eq., the final result can be written as

$$\sum U_{1-2} = \tfrac{1}{2}mv_2^2 - \tfrac{1}{2}mv_1^2$$

This equation represents the principle of work and energy for the particle. The term on the left is the sum of the work done by all the forces acting on the particle as the particle moves from point 1 to point 2. The two terms on the right side, which are of the form $T = \frac{1}{2} mv^2$, define the particle's final and initial kinetic energy, respectively. Like work, kinetic energy is a scalar and has units of joules (J) and ft · lb. However, unlike work, which can be either positive or negative, the kinetic energy is always positive, regardless of the direction of motion of the particle. it is often expressed in the form

$$T_1 + \Sigma U_{1-2} = T_2$$

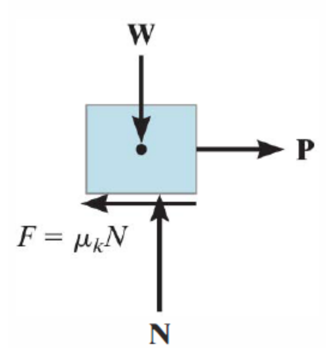
which states that the particle's initial kinetic energy plus the work done by all the forces acting on the particle as it moves from its initial to its final position is equal to the particle's final kinetic energy. As noted from the derivation, the principle of work and energy represents an integrated form of $\sum F_t = ma_t$, obtained by using the kinematic equation $a_t = v\,dv/ds$. As a result, this principle will provide a convenient substitution for $\sum F_t = ma_t$ when solving those types of kinetic problems which involve force, velocity, and displacement since these quantities are involved in above Eq. For application, it is suggested that the following procedure be used.

Work of Friction Caused by Sliding

A special class of problems will now be investigated which requires a careful application of Eq.

$$\Sigma T_1 + \Sigma U_{1-2} = \Sigma T_2$$

These problems involve cases where a body slides over the surface of another body in the presence of friction. Consider, for example, a block which is translating a distance s over a rough surface as shown in Fig. a.

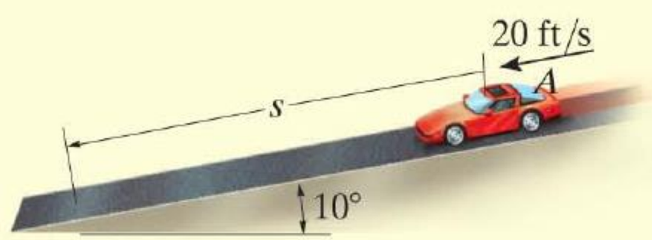


If the applied force P just balances the resultant frictional force $\mu_k.N$, Fig.b then due to equilibrium a constant velocity v is maintained, and one would expect above Eq. to be applied as follows:

$$\frac{1}{2}mv^2 + Ps - \mu_k Ns = \frac{1}{2}mv^2$$

Examples -1-

The 3500-lb automobile shown in Fig. a travels down the 10° inclined road at a speed of 20 ft/s. If the driver jams on the brakes, causing his wheels to lock, determine how far s the tires skid on the road. The coefficient of kinetic friction between the wheels and the road is $\mu_k = 0.5$.



(a)

SOLUTION

This problem can be solved using the principle of work and energy, since it involves force, velocity, and displacement.

Work (Free-Body Diagram). As shown in Fig. 14–10*b*, the normal force $\mathbf{N}_A$ does no work since it never undergoes displacement along its line of action. The weight, 3500 lb, is displaced s sin 10° and does positive work. Why? The frictional force $\mathbf{F}_A$ does both external and internal work when it undergoes a displacement s. This work is negative since it is in the opposite sense of direction to the displacement. Applying the equation of equilibrium normal to the road, we have

$$+\nwarrow \Sigma F_n = 0; \quad N_A - 3500 \cos 10^\circ \text{ lb} = 0 \qquad N_A = 3446.8 \text{ lb}$$

Thus,

$$F_A = \mu_k N_A = 0.5\,(3446.8 \text{ lb}) = 1723.4 \text{ lb}$$

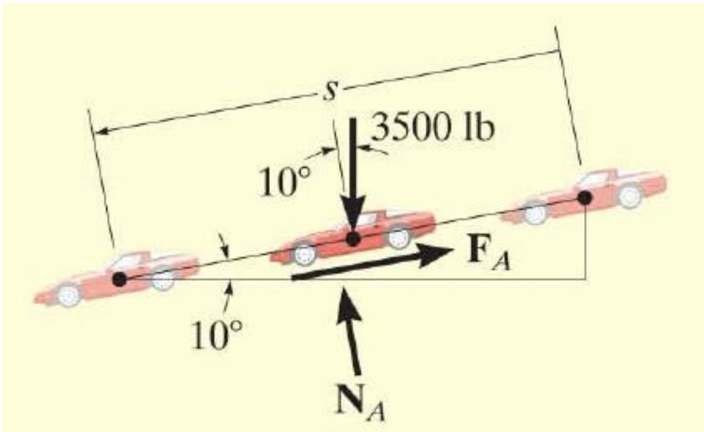
Principle of Work and Energy.

$$T_1 + \Sigma U_{1-2} = T_2$$

$$\frac{1}{2}\left(\frac{3500 \text{ lb}}{32.2 \text{ ft/s}^2}\right)(20 \text{ ft/s})^2 + 3500 \text{ lb}(s \sin 10^\circ) - (1723.4 \text{ lb})s = 0$$

Solving for s yields

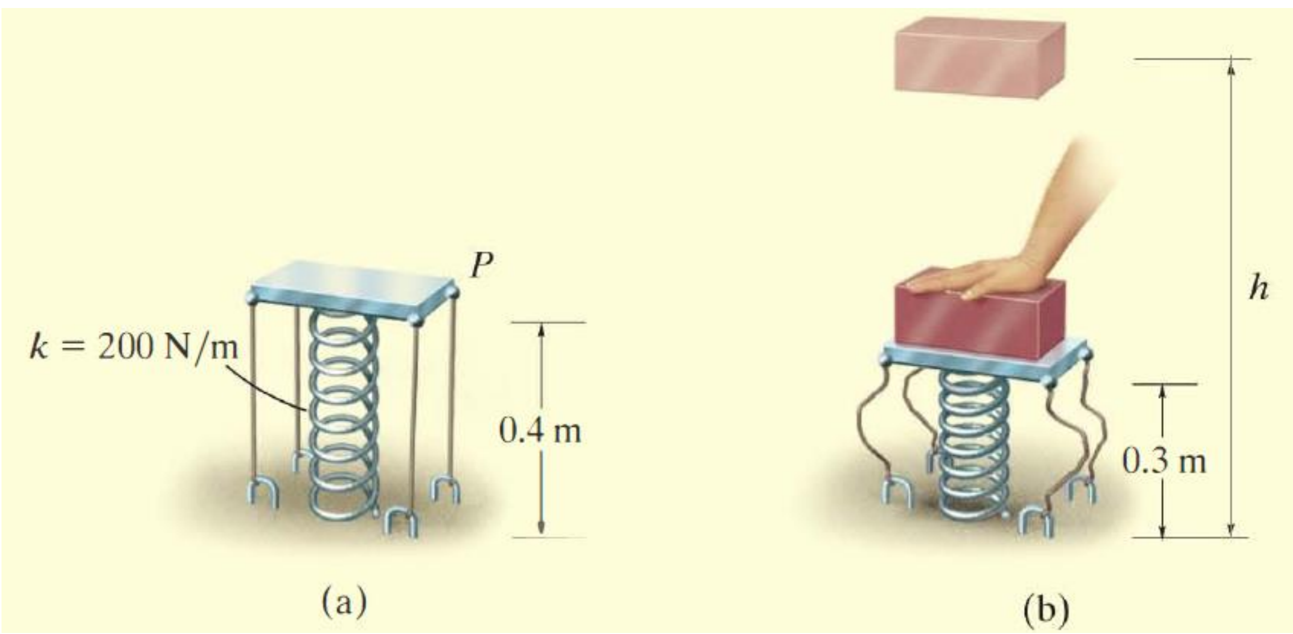
$$s = 19.5 \text{ ft} \qquad \textit{Ans.}$$



(b)

Examples -2-

The platform P, shown in Fig. a, has negligible mass and is tied down so that the O.4-m-Iong cords keep a I-m-Iong spring compressed 0.6 m when nothing is on the platform. If a 2-kg block is placed on the platform and released from rest after the platform is pushed down 0.1 m, Fig.b, determine the maximum height h the block rises in the air, measured from the ground .



(a) (b)

SOLUTION

Work (Free-Body Diagram). Since the block is released from rest and later reaches its maximum height, the initial and final velocities are zero. The free-body diagram of the block when it is still in contact with the platform is shown in Fig. 14–12*c*. Note that the weight does negative work and the spring force does positive work. Why? In particular, the *initial compression* in the spring is $s_1 = 0.6\text{ m} + 0.1\text{ m} = 0.7\text{ m}$. Due to the cords, the spring's *final compression* is $s_2 = 0.6\text{ m}$ (after the block leaves the platform). The bottom of the block rises from a height of $(0.4\text{ m} - 0.1\text{ m}) = 0.3\text{ m}$ to a final height h.

Principle of Work and Energy.

$$T_1 + \Sigma U_{1\text{-}2} = T_2$$

$$\tfrac{1}{2}mv_1^2 + \left\{-\left(\tfrac{1}{2}ks_2^2 - \tfrac{1}{2}ks_1^2\right) - W\,\Delta y\right\} = \tfrac{1}{2}mv_2^2$$

Note that here $s_1 = 0.7\text{ m} > s_2 = 0.6\text{ m}$ and so the work of the spring as determined from Eq. 14–4 will indeed be positive once the calculation is made. Thus,

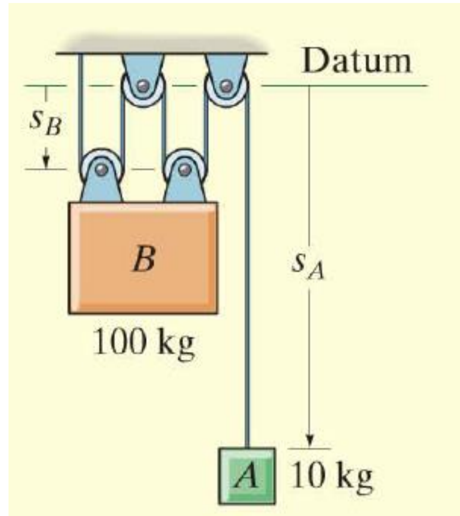
$$0 + \left\{-\left[\tfrac{1}{2}(200\text{ N/m})(0.6\text{ m})^2 - \tfrac{1}{2}(200\text{ N/m})(0.7\text{ m})^2\right] - (19.62\text{ N})[h - (0.3\text{ m})]\right\} = 0$$

Solving yields

$$h = 0.963\text{ m} \qquad \textit{Ans.}$$

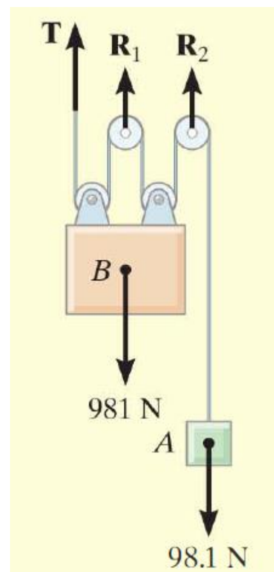
Examples -3-

Blocks A and B shown in Fig. a have a mass of 10 kg and 100 kg, respectively. Determine the distance B travels when it is released from rest to the point where its speed becomes 2 m/s.



(a)

Work (Free-Body Diagram). As shown on the free-body diagram of the system, Fig. b, the cable force T and reactions R 1 and R2 do no work, since these forces represent the reactions at the supports and consequently they do not move while the blocks are displaced. The weights both do positive work if we assume both move downward, in the positive sense of direction of SA and SB'



(b)

Principle of Work and Energy. Realizing the blocks are released from rest, we have

$$\Sigma T_1 + \Sigma U_{1-2} = \Sigma T_2$$

$$\left\{\tfrac{1}{2}m_A(v_A)_1^2 + \tfrac{1}{2}m_B(v_B)_1^2\right\} + \left\{W_A\,\Delta s_A + W_B\,\Delta s_B\right\} = \left\{\tfrac{1}{2}m_A(v_A)_2^2 + \tfrac{1}{2}m_B(v_B)_2^2\right\}$$

$$\{0 + 0\} + \{98.1\text{ N}\,(\Delta s_A) + 981\text{ N}\,(\Delta s_B)\} = \left\{\tfrac{1}{2}(10\text{ kg})(v_A)_2^2 + \tfrac{1}{2}(100\text{ kg})(2\text{ m/s})^2\right\} \qquad (1)$$

Kinematics. Using the methods of kinematics discussed in Sec. 12.9, it may be seen from Fig. 14–14*a* that the total length l of all the vertical segments of cable may be expressed in terms of the position coordinates s_A and s_B as

$$s_A + 4s_B = l$$

Hence, a change in position yields the displacement equation

$$\Delta s_A + 4\,\Delta s_B = 0$$

$$\Delta s_A = -4\,\Delta s_B$$

Here we see that a downward displacement of one block produces an upward displacement of the other block. Note that Δs_A and Δs_B must have the *same* sign convention in both Eqs. 1 and 2. Taking the time derivative yields

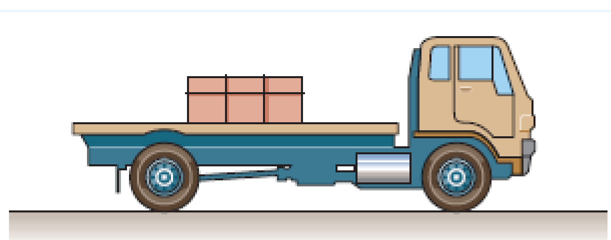
$$v_A = -4v_B = -4(2\text{ m/s}) = -8\text{ m/s} \qquad (2)$$

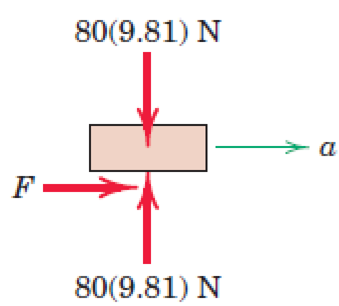
Retaining the negative sign in Eq. 2 and substituting into Eq. 1 yields

$$\Delta s_B = 0.883\text{ m}\downarrow \qquad \textit{Ans.}$$

Examples -4-

The flatbed truck, which carries an 80-kg crate, starts from rest and attains a speed of 72 km/h in a distance of 75 m on a level road with constant acceleration. Calculate the work done by the friction force acting on the crate during this interval if the static and kinetic coefficients of friction between the crate and the truck bed are





(a) 0.30 and 0.28, respectively, or (b) 0.25 and 0.20, respectively.

Solution. If the crate does not slip on the bed, its acceleration will be that of the truck, which is

$[v^2 = 2as]$ $$a = \frac{v^2}{2s} = \frac{(72/3.6)^2}{2(75)} = 2.67 \text{ m/s}^2$$

Case (a). This acceleration requires a friction force on the block of

$[F = ma]$ $$F = 80(2.67) = 213 \text{ N}$$

which is less than the maximum possible value of $\mu_s N = 0.30(80)(9.81) = 235$ N. Therefore, the crate does not slip and the work done by the actual static friction force of 213 N is

1 $[U = Fs]$ $$U_{1\text{-}2} = 213(75) = 16\,000 \text{ J} \quad \text{or} \quad 16 \text{ kJ}$$ *Ans.*

Case (b). For $\mu_s = 0.25$, the maximum possible friction force is $0.25(80)(9.81) = 196.2$ N, which is slightly less than the value of 213 N required for no slipping. Therefore, we conclude that the crate slips, and the friction force is governed by the kinetic coefficient and is $F = 0.20(80)(9.81) = 157.0$ N. The acceleration becomes

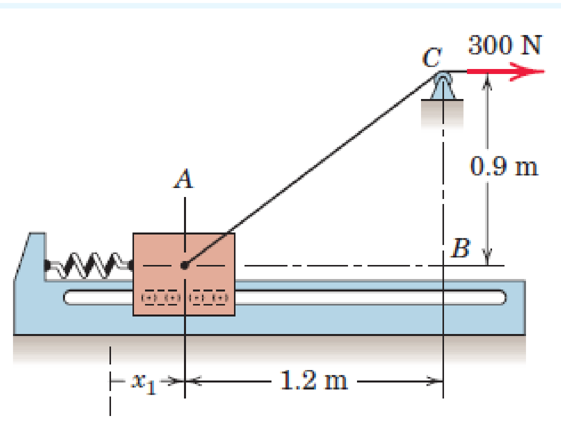
$[F = ma]$ $$a = F/m = 157.0/80 = 1.962 \text{ m/s}^2$$

The distances traveled by the crate and the truck are in proportion to their accelerations. Thus, the crate has a displacement of $(1.962/2.67)75 = 55.2$ m, and the work done by kinetic friction is

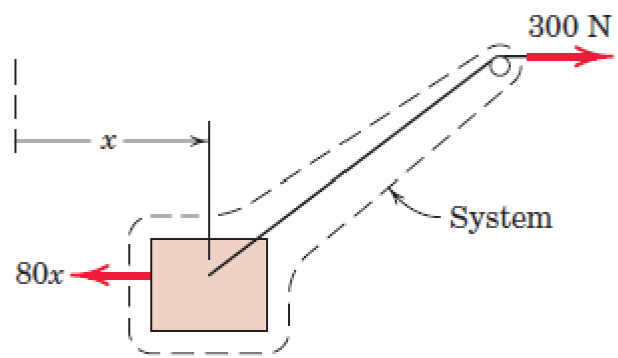
2 $[U = Fs]$ $$U_{1\text{-}2} = 157.0(55.2) = 8660 \text{ J} \quad \text{or} \quad 8.66 \text{ kJ}$$ *Ans.*

Examples -5-

The 50-kg block at A is mounted on rollers so that it moves along the fixed horizontal rail with negligible friction under the action of the constant 300-N force in the cable. The block is released from rest at A, with the spring to which it is attached extended an initial amount $x_1 = 0.233$ m. The spring has a



stiffness k = 80 N/m. Calculate the velocity v of the block as it reaches position B.



Solution. It will be assumed initially that the stiffness of the spring is smal enough to allow the block to reach position B. The active-force diagram for the system composed of both block and cable is shown for a general position. The spring force $80x$ and the 300-N tension are the only forces external to this system which do work on the system. The force exerted on the block by the rail, the weight of the block, and the reaction of the small pulley on the cable do no work on the system and are not included on the active-force diagram.

As the block moves from $x_1 = 0.233$ m to $x_2 = 0.233 + 1.2 = 1.433$ m, the work done by the spring force acting on the block is

$$[U_{1\text{-}2} = \frac{1}{2}k(x_1{}^2 - x_2{}^2)] \quad U_{1\text{-}2} = \frac{1}{2}80[0.233^2 - (0.233 + 1.2)^2]$$

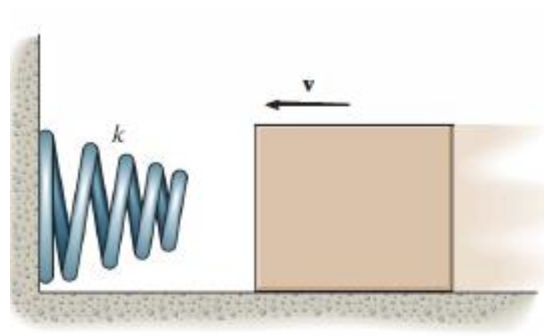
$$= -80.0 \text{ J}$$

The work done on the system by the constant 300-N force in the cable is the force times the net horizontal movement of the cable over pulley C, which is $\sqrt{(1.2)^2 + (0.9)^2} - 0.9 = 0.6$ m. Thus, the work done is $300(0.6) = 180$ J. We now apply the work-energy equation to the system and get

$$[T_1 + U_{1\text{-}2} = T_2] \quad 0 - 80.0 + 180 = \frac{1}{2}(50)v^2 \qquad v = 2.00 \text{ m/s} \qquad \textit{Ans}$$

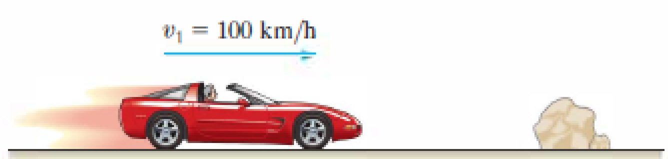
Problems

Q1/ The 1 .5-kg block slides along a smooth plane and strikes a nonlinear spring with a speed of v = 4 m/s. The spring is termed "nonlinear" because it has a resistance of $F_s = ks^2$, where $k = 900\ N/m^2$. Determine the speed of the block after it has compressed the spring s = 0.2 m.

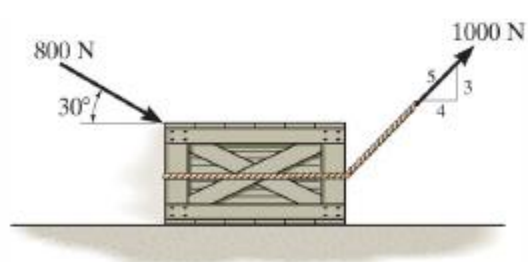


Q2/ The spring in the toy gun have an unscratched length of 100 mm. It is compressed and locked in the position shown. When the trigger is pulled, the spring outstretches 12.5 mm, and the 20-g ball moves along the barrel. Determine the speed of the ball when it leaves the gun. Neglect friction.

Q3/ The 2-Mg car has a velocity of V_1 = 100 km/h when the driver sees an obstacle in front of the car. It takes 0.75 s for him to react and lock the brakes, causing the car to skid. If the car stops when it has traveled a distance of 175 m, determine the coefficient of kinetic friction between the tires and the road.



Q4/ The crate, which has a mass of 100 kg, is subjected to the action of the two forces. If it is originally at rest, determine the distance it slides in order to attain a speed of 6 m/s. The coefficient of



kinetic friction between the crate and the surface is $\mu_k = 0.2$.

Q5/ If the 75-kg crate starts from rest at A, and its speed is 6 m/s when it passes point B, determine the constant force F exerted on the cable. Neglect friction and the size of the pulley.